Fuzzy Modeling Based on Generalized Conjunction Operations

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***Abstract*—A novel approach to fuzzy modeling based on the tuning of parametric conjunction operations is proposed. First, some novel methods for the construction of parametric generalized conjunction operations simpler than the known parametric classes of conjunctions are considered and discussed. Second, several examples of function approximation by fuzzy models, based on the tuning of the parameters of the new conjunction operations, are given and their approximation performances are compared with the approaches based on a tuning of membership functions and other approaches proposed in the literature. It is seen that the tuning of the conjunction operations can be used for obtaining fuzzy models with a sufficiently good performance when the tuning of membership functions is not possible or not desirable.**

***Index Terms*—Conjunction, disjunction, functions approximation, fuzzy modeling, t-norm.**

I. INTRODUCTION

FUZZY inference systems are known as universal approximators [13], [16], [24]. This property enables one to construct optimal fuzzy models of processes and systems. The optimization of such a fuzzy model is usually based on a tuning of the membership functions used in the model. However, we can point out at least two situations when such tuning may not be desirable or effective. The first situation usually arises when the initial shapes of the membership functions are based on some expert knowledge available about the system to be modeled. In such a case, after tuning of membership functions, this knowledge can be lost. The second situation arises when there exist limitations on the number of membership functions and rules used in the fuzzy model. These limitations may have an influence on the performance of fuzzy models. In such situations, the construction of optimal fuzzy models may also be based on the tuning of fuzzy connectives used in fuzzy model, if these connectives are given parametrically. Such tuning may be used instead of or additionally to tuning of the parameters of membership functions.

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The simplest of the fuzzy models may consist of the following types of fuzzy rules:

$$R_i: \text{If } X_1 \text{ is } A_{i1} \text{ and } X_2 \text{ is } A_{i2} \text{ and} \cdots \text{and } X_m \text{ is } A_{im}, \text{ then } Z_i = C_i \quad \text{(Mamdani model)}$$

$$R_i: \text{If } X_1 \text{ is } A_{i1} \text{ and } X_2 \text{ is } A_{i2} \text{ and} \cdots \text{and } X_m \text{ is } A_{im}, \text{ then } z_i = f_i(x_1, x_2, \ldots, x_m) \quad \text{(Sugeno model)}$$

where X_j and Z_i are fuzzy input and output variables (e.g., PRESSURE, VOLUME, TEMPERATURE, etc), A_{ij} and C_i are linguistic terms (e.g., SMALL, POSITIVE LARGE, ZERO), x_j and z_i are the real values of input and output variables and f_i are real valued functions. For given crisp input values x_j^*, the firing values of the rules may be calculated as follows:

$$w_i = T_{im-1}\left(\ldots T_{i2}\left(T_{i1}\left(\mu_{Ai1}(x_1^*), \mu_{Ai2}(x_2^*)\right), \mu_{Ai3}(x_3^*)\right), \ldots, \mu_{Aim}(x_m^*)\right)$$

where T_{ij} are some conjunction operations (usually identical t-norms for all i and j) used as the connective *and* and $\mu_{Aij}(x_j^*)$ are the membership values of x_j^* in fuzzy sets A_{ij}. After aggregation and defuzzification of conclusions of rules (these methods usually differ for different types of models and methods [11], [13], [16], [19], [22], [24]), the output of fuzzy model will be obtained as some real value z^*. As a result, fuzzy models determine some real valued functions $z^* = z(x_1^*, x_2^*, \ldots, x_m^*)$ which may be used as an approximation of the given experimental data or as an approximation of traditional mathematical model of some system or process.

The membership functions in fuzzy models are often given parametrically and a tuning of these parameters is used for the minimization of the approximation error. The use of parametric conjunction operations T_{ij} in a fuzzy model gives rise to the possibility of tuning the parameters of this operation. Unfortunately, the known parametric classes of t-norms and t-conorms, which can be used in fuzzy models, are generally too complicated for tuning and for hardware realization. The construction of simpler parametric classes of these operations, therefore, looks very attractive. One possible way of such simplification may be based on the deletion of the associativity property and, perhaps, the commutativity property from the definition of conjunction and disjunction operations [2], [3]. It is clear that the associativity property is not necessary for conjunction operations used in fuzzy models with two inputs and one output. For rules with a greater number of inputs and outputs these two properties are also not necessary if the sequence of operations and procedures used for processing fuzzy rules are fixed. Moreover, the noncommutativity of conjunction may in fact be desirable for rules considered above because it can reflect different influences of the input variables on the output of the system. For example, in fuzzy control, the positions of the input variables the "error" and the "change in error" in rules are usually fixed and these variables have different influences on the output of the system. In the application areas of fuzzy models when the sequence of operands is not fixed, the property of noncommutativity may not be desirable. In such cases, the parametric operations satisfying the commutativity property or the generalized commutativity property as introduced in this paper may be used.

The influence of fuzzy operations on the behavior of fuzzy systems and the tuning of operations in fuzzy models have been considered in [3], [5], [7], [8], [11], [14], [18], [19], [23], and [26]. The methods of generation of nonassociative and noncommutative conjunction and disjunction operations have been introduced in [2]–[4].

This paper dwells upon the general class of fuzzy conjunctions introduced in [4] and some theoretical results describing the methods of generation of these operations are given. Examples of fuzzy modeling based on the optimization of the new operations are compared with the methods reported in literature that are based on an optimization of the membership functions. It is shown that the tuning of the parameters of generalized fuzzy conjunction operations can be used for construction of fuzzy model and functions approximation.

II. Parametric Conjunction and Disjunction Operations

Traditionally, t-norms T and t-conorms S are considered as conjunction and disjunction operations respectively. These operations are defined as functions $T, S : [0,1]\times[0,1] \to [0,1]$ satisfying the monotonicity, the commutativity, the associativity properties, and the boundary conditions: $T(x,1) = x$, $S(x,0) = x$, respectively [1], [15], [25]. The function $N : [0,1] \to [0,1]$ satisfying the properties: $N(0) = 1$, $N(1) = 0$, $N(x) \le N(y)$ if $y \le x$, is called negation. T-norms and t-conorms can be obtained one from another as follows: $S(x,y) = N(T(N(x),N(y)))$ and $T(x,y) = N(S(N(x),N(y)))$, where N is an involutive negation, i.e., N satisfies the property $N(N(x)) = x$. We will use further the following simplest examples of t-norms and t-conorms, mutually related by means of involutive negation $N(x) = 1 - x$:

$$T_c(x,y) = \min\{x,y\} \quad S_c(x,y) = \max\{x,y\}$$
$$T_p(x,y) = xy \quad S_p(x,y) = x + y - xy$$
$$T_b(x,y) = \max\{0,(x+y-1)\}$$
$$S_b(x,y) = \min\{1,(x+y)\}.$$

From the associativity property it follows that t-norms can be generated by some inverse function, particularly, as follows: $T(x,y) = \varphi^{-1}(T_1(\varphi(x),\varphi(y)))$, where φ is some increasing bijection $\varphi : [0,1] \to [0,1]$ with $\varphi(0) = 0$, $\varphi(1) = 1$ and T_1 is some t norm, for example, one of the previously considered. The known parametric classes of t-norms are generally too complicated for tuning and hardware realization. The simplest parametric classes of t-norms are the following:

$$T(x,y) = 1 - \sqrt[p]{(1-x)^p + (1-y)^p - (1-x)^p(1-y)^p}$$
(Schweizer and Sklar)
$$T(x,y) = 1 - \min\left(1, \sqrt[p]{(1-x)^p + (1-y)^p}\right).$$
(Yager)

The complexity of the parametric classes of t-norms such as the ones previously given is due to the associativity property that requires using the inverse function φ^{-1} for the generation of these functions.

The way of generation of simpler parametric classes of conjunction and disjunction operations is introduced in [2] and [3]. These operations are defined as functions $T, S : [0,1]\times[0,1] \to [0,1]$ satisfying on [0, 1] the following conditions:

$$T(x,1) = T(1,x) = x$$
$$S(x,0) = S(0,x) = x \quad \text{(boundary conditions)} \tag{1}$$
$$T(x,y) \le T(u,v)\ if\ x \le u, y \le v$$
$$S(x,y) \le S(u,v)\ if\ x \le u, y \le v \quad \text{(monotonicity)}. \tag{2}$$

It is clear that these operations satisfy the following properties:

$$T(0,0) = T(0,1) = T(1,0) = 0$$
$$S(0,1) = S(1,0) = S(1,1) = 1 \tag{3}$$
$$T(1,1) = 1 \quad S(0,0) = 0. \tag{4}$$
$$T(x,0) = T(0,y) = 0 \quad S(1,y) = S(x,1) = 1. \tag{5}$$

Conjunction and disjunction operations may also be obtained one from another by means of involutive negation N : $S(x,y) = N(T(N(x),N(y)))$ and $T(x,y) = N(S(N(x),N(y)))$. In [3], some methods for the generation of parametric classes of conjunction operations are described. The approach may be formulated as follows.

Theorem 1: Suppose T_1, T_2 are conjunctions, S_1 and S_2 are functions $S_i : [0,1]\times[0,1] \to [0,1]$ satisfying (2) and (3), the functions $h, g_1, g_2 : [0,1] \to [0,1]$ are nondecreasing such that $g_1(1) = g_2(1) = 1$; then, the following functions:

$$T(x,y) = T_2\left(T_1(x,y), S_1\left(g_1(x), g_2(y)\right)\right) \tag{6}$$
$$T(x,y) = T_2\left(T_1(x,y), g_1\left(S_1(x,y)\right)\right) \tag{7}$$
$$T(x,y) = T_2\left(T_1(x,y), S_2\left(h(x), S_1(x,y)\right)\right) \tag{8}$$

are conjunctions.

The functions g and h are called generators. As conjunctions T_2 and T_1 and function S_1 one can use, for example, the simplest t-norms T_c, T_p, T_b, and t-conorms S_c, S_p, S_b. Disjunction operations may be generated dually or obtained from conjunctions by means of negation operation. Examples of simple parametric conjunctions obtained in this way with generators $g(x) =$

$\max\{1-p(1-x),0\}, g(x)=x^q, h(x)=t, (p,q\geq, 0\leq t\leq 1)$ are the following:

$$T(x,y) = (xy)\max\{1-p(1-x), y^q\} \quad (9)$$
$$T(x,y) = \min(x,y)(x^p+y^q-x^py^q) \quad (10)$$
$$T(x,y) = \min(x,y)\max(x^p,y^q) \quad (11)$$
$$T(x,y) = \min(x,y)(x+y-xy)^p \quad (12)$$
$$T(x,y) = (xy)\max\{t,(x+y-xy)^p\}. \quad (13)$$

It is clear that conjunctions (6)–(8) are commutative if T_1, T_2, and S_1 are commutative and additionally if $g_1=g_2$ in (6) and $h(x)=t$, where t is a constant such that $0\leq t\leq 1$, in (8). As one can see, the conjunctions (12) and (13) are commutative. The conjunctions (10) and (11) also became commutative when $p=q$. The permutation of x and y in the left-hand sides of (10) and (11) will change the value of $T(x,y)$ when p does not equal to q. However, we can permute x^p and y^q in the right-hand sides of conjunctions (10) and (11) and the value of correspondent conjunction $T(x,y)$ will be not changed. Such kind of "commutativity" of conjunction operations will be called generalized commutativity of parametric conjunctions and disjunctions. More exactly, we will say that conjunction (6) *satisfies the property of generalized commutativity* if T_1 and S_1 are commutative, $g(x,p)$ is a generator dependent on parameter p, and $g_1(x)=g(x,p_x)$, $g_2(y)=g(y,p_y)$ where p_x and p_y are the values of parameter p. The conjunctions (10) and (11) satisfy the property of generalized commutativity, but the conjunction (9) does not satisfy this property.

A tuning of the parametric conjunctions satisfying the property of generalized commutativity may be started with equal values of the parameters p_x and p_y. In such a case, this conjunction will be commutative and will not depend on the order of its operands. After a separate tuning of the parameters, the values p_x and p_y reached will reflect the influence of these parameters on the performance of the fuzzy model. The example of function approximation based on the optimization of the parameters of the conjunction (10) satisfying the generalized commutativity conditions is considered in [3].

III. G-Conjunction and G-Disjunction Operations

In this section, a more general definition of conjunction and disjunction operations is considered which enables the building of simpler parametric conjunction operations. These generalized operations will be called G-conjunction and G-disjunction, respectively.

Definition 1: Functions $T,S:[0,1]\times[0,1]\to[0,1]$ are a G-conjunction and a G-disjunction, respectively, if they satisfy the conditions (2)–(4).

It follows from (2) and (3) that G connectives satisfy also the relation (5). The conditions (2)–(4) are considered in [17] as axiomatic skeletons for the intersection and the union of fuzzy sets. The most general definition of conjunction and disjunction operations may be based only on the axioms (3) and (4) [9].

Theorem 2: Suppose N is a negation on [0, 1] and T, S are some G-conjunction and G-disjunction, respectively; then the following relations define, correspondingly, G-disjunction and G-conjunction operations:

$$S_T(x,y) = N(T(N(x),N(y)))$$
$$T_S(x,y) = N(S(N(x),N(y))).$$

Proof: $S_T(0,0)=N(T(N(0),N(0)))=N(T(1,1))=N(1)=0$. $S_T(x,1)=N(T(N(x),N(1)))=N(T(N(x),0))=N(0)=1$. Similarly, we obtain $S_T(1,x)=1$. The monotonicity property of S_T follows from monotonicity of T and N. Proof for T_S is similar.

It follows from Theorem 2 that by means of any noninvolutive negation [6] it is possible to obtain some G-conjunction or G-disjunction from the conjunctions and the disjunctions considered in Section II. However, such an approach does not result in construction of simple operations.

Corollary: If N is an involutive negation, then for any G-conjunction T and G-disjunction $S=S_T$, and for any G-disjunction S and G-conjunction $T=T_S$, the following De Morgan laws are fulfilled:

$$N(S(x,y)) = T(N(x),N(y))$$
$$N(T(x,y)) = S(N(x),N(y)).$$

Of course, at first sight, it seems unnatural that the boundary conditions (1) are absent in the definition of fuzzy connectives. However, there exist some rational explanations for the situations $T(x,1)\leq x$ and $y\leq T(y,1)$ for some x and y from [0, 1]. The first is used in the definition of a weak t-norm [9]. The second can be fulfilled when the conjunction possesses some features of the aggregation operation [17]. In any case, by weakening the restrictions on conjunctions and disjunctions it is possible to build more flexible functions that can result in a better performance of fuzzy models. The following statement paves the way for the generation of new operations.

Theorem 3: Suppose T_1 is a G-conjunction, S_1 is a G-disjunction and f, g, $h:[0,1]\to[0,1]$ are nondecreasing functions such that $f(0)=g(0)=h(0)=0$, $f(1)=g(1)=h(1)=1$; then, the functions

$$T(x,y) = f(T_1(g(x),h(y)))$$
$$S(x,y) = f(S_1(g(x),h(y)))$$

are a G-conjunction and a G disjunction, respectively.

Proof: $T(0,y)=f(T_1(g(0),h(y)))=f(T_1(0,h(y)))=f(0)=0$. Similarly, $T(y,0)=0$. $T(1,1)=f(T_1(g(1),h(1)))=f(T_1(1,1))=f(1)=1$. The monotonicity of T follows from the monotonicity of T_1, f, g, and h. The proof of the properties (2)–(4) for S is similar.

The functions f, g and h used in the generation of G-conjunctions and G-disjunctions are called generators of these operations. It is clear that G-conjunctions and G-disjunctions defined in Theorem 3 are commutative if $g=h$ and T_1, S_1 are, respectively, commutative. In a similar way as done to the conjunctions considered in the previous sections, we will say that these functions satisfy the property of generalized commutativity if T_1 and S_1 are commutative, $g_1(x,p)$ is a generator dependent on parameter p, and $g(x)=g_1(x,p_x)$, $h(y)=g_1(y,p_y)$ where p_x and p_y are the values of parameter p.

Theorem 1 may be extended on G connectives as follows.

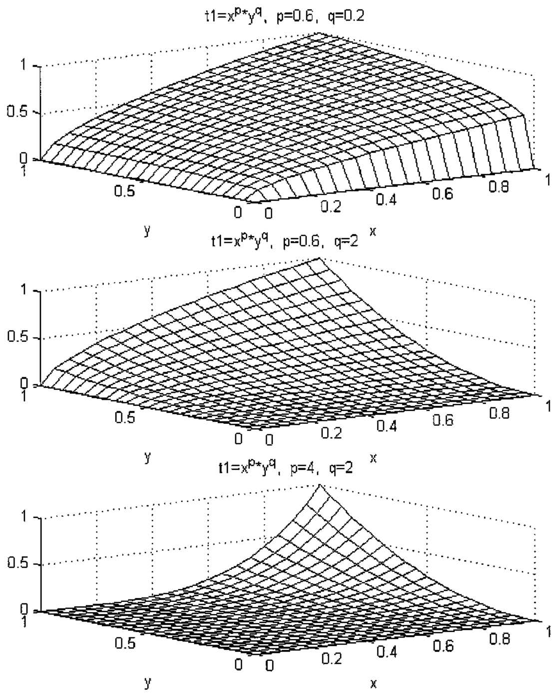

Fig. 1. The surfaces of the conjunction operation $T(x, y) = x^p y^q$ for different values of the parameters p and q.

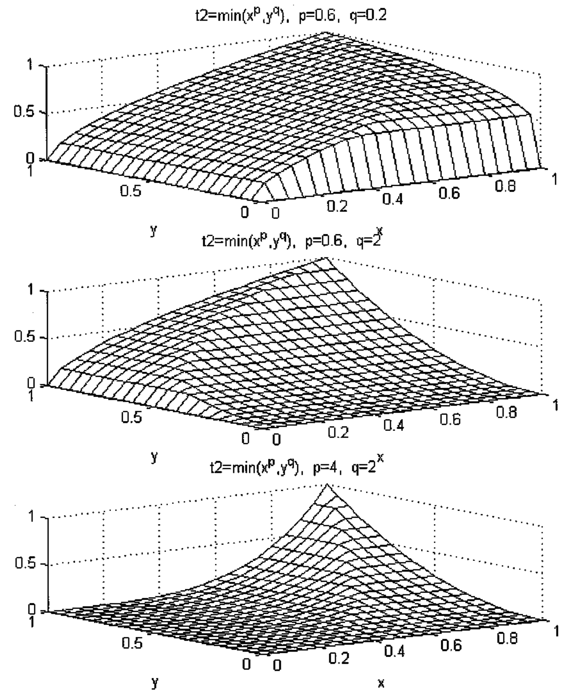

Fig. 2. The surfaces of the conjunction operation $T(x, y) = \min(x^p, y^q)$ for different values of the parameters p and q.

Theorem 4: Suppose T_1 and T_2 are G-conjunctions, S_1 and S_2 are functions $S_i : [0,1] \times [0,1] \to [0,1]$ satisfying (2) and (3), h, g_1, $g_2 : [0,1] \to [0,1]$ are nondecreasing functions such that $g_1(1) = g_2(1) = 1$; then, the following functions:

$$T(x,y) = T_2\left(T_1(x,y), S_1\left(g_1(x), g_2(y)\right)\right)$$
$$T(x,y) = T_2\left(T_1(x,y), g_1\left(S_1(x,y)\right)\right)$$
$$T(x,y) = T_2\left(T_1(x,y), S_2\left(h(x), S_1(x,y)\right)\right)$$

are G-conjunctions.

Proof: The monotonicity of T in all expressions follows from the monotonicity of functions used in the right-hand sides of the expressions. If $x = 0$ or $y = 0$, then $T_1(x,y) = 0$ and, hence, $T(x,y) = T_2(0,z) = 0$. If $x = y = 1$, then $T_1(1,1) = 1$, all S_1 and g_1 equal to 1, from (5) it follows also that $S_2 = 1$ and, hence, in all expressions we have $T(x,y) = T_2(1,1) = 1$.

By the use of Theorems 3 and 4, the simplest parametric G-conjunction operations can be obtained

$$T(x,y) = x^p y^q \tag{14}$$
$$T(x,y) = \min\left(x^p, y^q\right)$$
$$T(x,y) = (xy)^p (x + y - xy)^q \tag{15}$$

where $p, q \geq 0$. The surfaces of these conjunctions are shown in Figs. 1–3. The last G-conjunction is commutative and the first two G-conjunctions satisfy the property of generalized commutativity.

It is seen that the proposed definition of conjunction and disjunction operations enables one to build the simplest parametric classes of conjunction and disjunction operations. It is to be noted that another system of axioms for generalized connectives is considered in [7] where the fuzzy conjunction $(xy)^k$ is considered and its application to fuzzy modeling is discussed. This conjunction belongs to the parametric class of conjunctions (14) with $p = q$. In the following sections, we discuss two examples of function approximations by fuzzy models based on the tuning of new conjunctions.

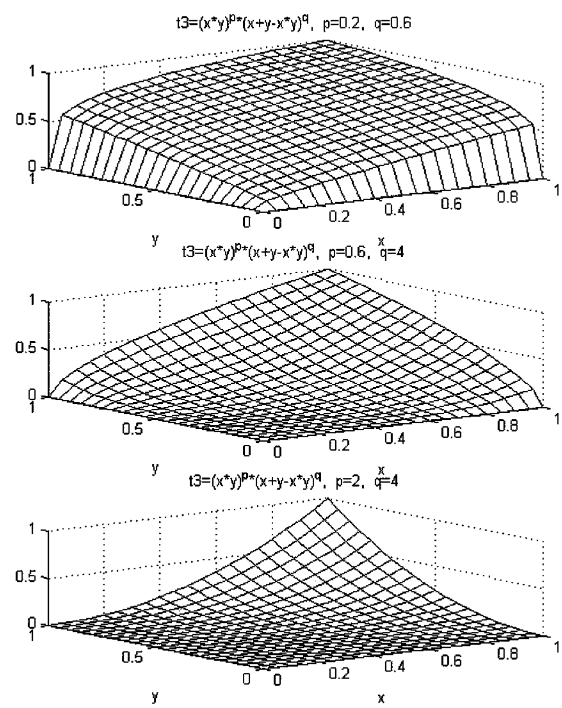

Fig. 3. The surfaces of the conjunction operation $T(x, y) = (xy)^p(x + y - xy)^q$ for different values of the parameters p and q.

IV. Approximation of Two-Input sinc Function

First, we consider the example of approximation of a two-input sinc function $z = \mathrm{sinc}(x,y) = \sin(x)\sin(y)/xy$, with x

and y defined on [−10, 10] by a Sugeno fuzzy model. This example is considered in [13] where 121 values of function (obtained for input variables x and y evenly distributed on [−10, 10]) are used for the approximation. The adaptive neuro-fuzzy inference system (ANFIS) used contains 16 rules with four generalized bell membership functions assigned to each input variable. The total number of fitting parameters is, therefore, 72, including 24 premise parameters (three parameters for each fuzzy set) and 48 consequent parameters.

In our simulations, sinc function is represented by 441 values calculated for all integer values $\{-10, -9, \ldots, 9, 10\}$ of the input variables x and y. We have used a Sugeno fuzzy model with nine rules containing three triangular membership functions defined on the domains of each input variable. These three membership functions have the value 1 for the values of input variables equal to −10, 0, and 10, respectively. The correspondent fuzzy sets are denoted as N (negative), Z (zero) and P (positive). Considering all possible combinations of the fuzzy sets A_i and B_i defined by these membership functions, we arrive at the Sugeno model that consists of the following nine rules:

$$R_i: \text{If } X \text{ is } A_i \text{ and } Y\ is\ B_i \text{ then } z = r_i x + s_i y + t_i \quad (i = 1, 2, \ldots, 9)$$

where r_i, s_i, and t_i are real valued parameters. In our simulations the parametric G-conjunction operations $T(u,v) = u^{p_i} \cdot v^{q_i}$ were used as *and* operation in the rules R_i of this fuzzy model. For given real values of x and y the output of the fuzzy model is calculated as

$$z^* = \frac{\sum_{i=1}^{9} w_i z_i}{\sum_{i=1}^{9} w_i}$$

where $z_i = r_i x + s_i y + t_i$, $w_i = (\mu_{A_i}(x))^{p_i} \cdot (\mu_{B_i}(y))^{q_i}$ and A_i and B_i, ($i = 1, 2, \ldots, 9$), are fuzzy sets from rules R_i defined on X and Y, respectively. The tuning of the 18 parameters of operations and 27 parameters of right sides of rules was done for minimization of the approximation error

$$E = \sqrt{\frac{\sum_{k=1}^{N} \left(z^{(k)} - z^{*(k)}\right)^2}{N}}$$

where $N = 441$ is a number of approximated values of sinc function, $z^{(k)}$ and $z^{*(k)}$ are kth values of sinc function and fuzzy model, respectively, in the given point ($x^{(k)}$, $y^{(k)}$). The minimization of approximation error was based on the use of the sequential-quadratic programming method [10], [20], [21]. This method was used for solution the constraint nonlinear optimization problem with constraints $0.0002 \leq p_i, q_i \leq 100$ and $-80 \leq r_i, s_i, t_i \leq 80$ ($i = 1, 2, \ldots, 9$). The initial values of all parameters were equal to 1. All membership functions were kept fixed during the optimization process. The shapes of the sinc function and the fuzzy model obtained after tuning of parameters are shown on Fig. 4. The error of approximation is equal to 0.046 and better than the result (RMSE $\approx$ 0.1) obtained after the training of a multilayer perceptron (MLP) with 73 fitting parameters (connection weights and thresholds) reported in [13]. The error of approximation of 121 values of the sinc function obtained by ANFIS with 16 rules and 72 fitting parameters [13] is less than that obtained in our simulations. However, as is seen from Fig. 4, the results of function approximation by fuzzy model may be considered as sufficiently good, taking into account that the number of rules and parameters used in model are equal to 9 and 45, respectively, and number of approximated points equals to 441.

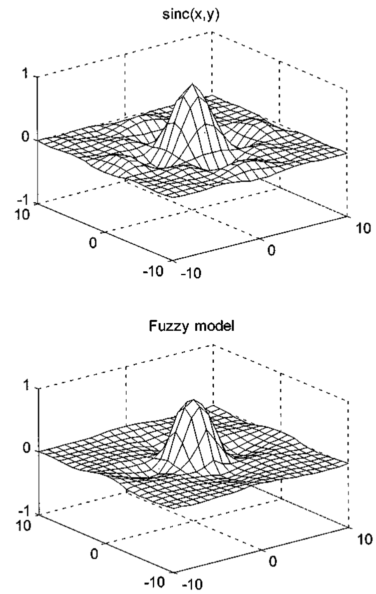

Fig. 4. The results of the approximation of two-input sinc function by a fuzzy model based on a tuning of the parameters of the G-conjunction $T(u,v) = u^p v^q$.

V. Approximation of Three-Input Nonlinear Function

As another example, we have approximated the following three-input nonlinear function given:

$$f(x,y,z) = \left(1 + \sqrt{x} + \frac{1}{y} + \frac{1}{z\sqrt{z}}\right)^2. \tag{16}$$

This function, defined on [1, 6] × [1, 6] × [1, 6], has been used in several papers [12], [13] for testing different modeling approaches. In our studies, we have used (as in [13]) 216 training data and 125 checking data, sampled uniformly from the input ranges [1, 6] × [1, 6] × [1, 6] and [1, 5] × [1, 5] × [1, 5], respectively. As *and* connectives in our fuzzy model we have used the simplest G-conjunctions $T(u,v) = u^p v^q$, extended to three parameters. The approximation results obtained (which are not given here due to the lack of space) indicate better performance both on the training and the checking data, in comparison with the results obtained by different authors and reported in literature [12], [13].

VI. Conclusion

The construction of simple parametric classes of fuzzy connectives for fuzzy modeling is discussed in this paper. For this purpose, the generalization of the known definitions of the conjunction and disjunction operations is considered. A novel approach to the definition and the generation of generalized conjunction and disjunction operations is proposed.

This approach is based on the deletion of the associativity and the commutativity properties from the definition of fuzzy conjunction and disjunction operations. Nevertheless, the proposed method gives the possibility of generating commutative operations. The concept of generalized commutativity for parametric conjunction operations is introduced and discussed. Several examples of parametric classes of conjunctions that are simpler than the known parametric classes of conjunctions are given. Our simulations show that the tuning of parametric operations might be used for the identification of fuzzy models when the tuning of membership functions is impossible or not desirable, for example, for keeping the expert knowledge about the membership functions intact.

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