Sliding-Mode Control With Soft Computing: A Survey

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***Abstract*—Sliding-mode control (SMC) has been studied extensively for over 50 years and widely used in practical applications due to its simplicity and robustness against parameter variations and disturbances. Despite the extensive research activities carried out, the key technical problems associated with SMC remain as challenging research questions due to demands for new industrial applications and technological advances. In this respect, soft computing (SC) is a rather recent development in intelligent systems which has provided alternative means for adaptive learning and control to overcome the key SMC technical problems. Substantial efforts in integration of SMC with SC have been placed in recent years with various successes. In this paper, we provide the state of the art of recent developments in SMC systems with SC, examining key technical research issues and future perspectives.**

***Index Terms*—Chaos, chattering, evolutionary computation, fuzzy logic, neural networks, robustness, sliding-mode control, variable-structure systems.**

I. Introduction

SLIDING-MODE control (SMC) is a special class of the variable-structure systems (VSSs) [1], [2]. It has been studied extensively for over 50 years and widely used in practical applications due to its simplicity and robustness against parameter variations and disturbances [3], [4]. The essence of SMC is that in a vicinity of a prescribed switching manifold, the velocity vector of the controlled state trajectories always points toward the switching manifold. Such motion is induced by imposing disruptive (discontinuous) control actions, commonly in the form of switching control strategies. An ideal sliding mode exists only when the system state satisfies the dynamic equation that governs the sliding mode for all time. This requires an infinite switching, in general, to ensure the sliding motion.

Since its inception in late 1950s, a huge body of research has been devoted to SMC, as depicted by, for example, the papers such as [2] and [5] which surveyed the historic trends of research in this field in different periods. This is also evidenced by, for example, a simple *Google Scholar* search of the term "sliding mode control," which returned over 440 000 items (as of May 28, 2009).

Despite the sustained active research on SMC over the last 50 years, the key technical problems such as chattering, the removal of the effects of unmodeled dynamics, disturbances and uncertainties, adaptive learning, and improvement of robustness remain to be the key research challenges that have attracted continuing attention. Various approaches have been developed to address these problems, although there has not been a perfect solution. The integration of other research methodologies has been considered, and many new technologies have been introduced, such as those which will be discussed in this paper. One key objective of the recent SMC research is to make it more intelligent. Naturally, this leads to the introduction of intelligent agents into SMC paradigms.

Soft computing (SC, as opposed to hard computing), as coined by Prof. Lotif Zadeh as one of the key future intelligent systems technologies, has been researched and applied in solving various practical problems extensively. SC technologies include neural networks (NNs), fuzzy logic (FL), and probabilistic reasoning (PR) which are paradigms for mimicking human intelligence and smart optimization mechanisms observed in the nature [6]. The diffusion of SC technologies into SMC architectures has received numerous successes, and has become a major research topic in SMC theory and applications. This is certainly supported by *Google Scholar* searches (as of May 28, 2009) of the key words "sliding mode" combined with "fuzzy," "neural network," and "genetic algorithm," which returned over 28 000, 24 000, and 15 000 items, respectively. Despite this fast development, there has not been a comprehensive survey to examine the current progress of and the future perspectives on the integration of SMC with SC. There is a significant need to examine the state of the art of SMC with SC, and to look into future perspectives of research in this area which will see further significant applications in the years to come.

The integration of SMC and SC has two aspects: one is the application of SC technologies in SMC to alleviate the problems or the shortcomings of the classical SMC techniques, and the other is the use of SMC theory to enhance the capabilities of SC technologies. This survey will be focused on the application of SC technologies in SMC since this special issue is concentrated on SMC and its applications.

This paper is organized as follows. Section II presents the fundamentals of SMC theory and outlines the key technical issues associated with it. Section III briefs the basics of SC techniques that are relevant to SMC. Section IV surveys the state

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of the art of SMC with SC. Section V presents the challenges facing SMC with SC. Section VI concludes this paper.

II. SMC Theory

A. Fundamentals of SMC

The SMC theory was originated in late 1950s in the former USSR, led by Prof. V. I. Utkin and Prof. S. V. Emelyanov [2] to address specific problems associated with a special class of VSSs, which are the control systems involving discontinuous control actions.

In the early days of VSS, the research was focused on single-input and single-output systems, and various well-known methodologies were developed, e.g., the eigenvalue assignment approach [2], the Fillipov approach [5], etc. In recent years, the majority of research in SMC has been done with regard to multiinput and multioutput systems (MIMO), so we will use MIMO system framework as a platform for discussing the integration of SC methodologies in SMC. Readers can also refer to the comprehensive tutorial [7] on the fundamentals of MIMO SMC systems.

Commonly, the MIMO SMC systems considered are of the form

$$\dot{x} = f(x,t) + B(x,t)u + \xi(x,t) \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state vector $u \in \mathbb{R}^m$, and $\xi(x,t) \in \mathbb{R}^n$ represents all the factors that affect the performance of the control system, such as disturbances and uncertainties in the parameters of the system. If $\xi \in$ range B, then there exists a control u_ξ, such as $\xi = Bu_\xi$. It is well known that when this condition (the matching condition) is satisfied, the celebrated invariance property of SMC stands [8].

The design procedure of SMC includes two major steps encompassing two main phases of SMC:

1) Reaching phase: where the system state is driven from any initial state to reach the switching manifolds (the anticipated sliding modes) in finite time;
2) Sliding-mode phase: where the system is induced into the sliding motion on the switching manifolds, i.e., the switching manifolds become an attractor.

These two phases correspond to the following two main design steps.

1) Switching manifold selection: A set of switching manifolds are selected with prescribed desirable dynamical characteristics. Common candidates are linear hyperplanes.
2) Discontinuous control design: A discontinuous control strategy is formed to ensure the finite time reachability of the switching manifolds. The controller may be either local or global, depending upon specific control requirements.

In the context of the system (1), following the main SMC design steps, the switching manifolds can be denoted as $s(x) = 0 \in \mathbb{R}^m$, where $s = (s_1, \ldots, s_m)^{\mathrm{T}}$ is a m-dimensional vector underpinned by the desired dynamical properties.

The SMC $u \in \mathbb{R}^m$ is characterized by the control structure defined by

$$u_i = \begin{cases} u_i^+(x), & \text{for } s_i(x) > 0 \\ u_i^-(x), & \text{for } s_i(x) < 0 \end{cases} \tag{2}$$

where $i = 1, \ldots, m$.

According to the SMC theory, when the sliding mode occurs, a so-called "equivalent control" is induced, i.e.,

$$\dot{s} = \frac{\partial s}{\partial x}\dot{x} = \frac{\partial s}{\partial x}\left(f(x,t) + B(x,t)u + \xi(x,t)\right). \tag{3}$$

Without loss of generality, we assume that $(\partial s/\partial x)B(x,t)$ is nonsingular. It is commonly understood that in the sliding mode, there exists a virtual control signal u_{eq} which drives the system dynamics $\dot{s} = 0$, resulting in

$$u_{\mathrm{eq}} = -\left(\frac{\partial s}{\partial x}B(x,t)\right)^{-1}\left(\frac{\partial s}{\partial x}\left(f(x,t) + \xi(x,t)\right)\right). \tag{4}$$

When in the sliding mode, if $\xi(x,t)$ satisfies the matching condition, i.e., $\xi(x,t) = B(x,t)u_\xi$, the motion is then governed by

$$\dot{x} = \left[I - B(x,t)\left(\frac{\partial s}{\partial x}B(x,t)\right)^{-1}\frac{\partial s}{\partial x}\right]f(x,t) \tag{5}$$

which is immune to the external force $\xi(x,t)$. This is the essence of the celebrated invariant property of SMC [8].

B. SMC Design Methods

There are several SMC controller types seen in the literature, which one to use is dependent upon the specific problems to be dealt with. However, central to the SMC design is the use of the Lyapunov stability theory, in which the Lyapunov function of the form

$$V = \frac{1}{2}s^{\mathrm{T}}s \tag{6}$$

is commonly taken. The control design task then becomes to find a suitable discontinuous control such that $\dot{V} < 0$ in the neighborhood of the equilibrium.

If one would like to embed learning and adaptation in SMC to enhance its performance, one common alternative Lyapunov function may be constructed as

$$V = \frac{1}{2}s^{\mathrm{T}}Qs + \frac{1}{2}z^{\mathrm{T}}Rz \tag{7}$$

where $z = 0$ represents a desirable outcome and R and Q are symmetric nonnegative definite matrices of appropriate dimensions. For example, if one wants to enable system parameter learning while controlling, for a set of system parameters $p \in \mathbb{R}^l$ to converge to its true values p^*, one may set $z = p - p^*$. Therefore, $\dot{V} < 0$ in the neighborhood of $V = 0$ would lead to the convergence of the system states during which certain parameters would be learned simultaneously. However, it should be pointed out that the set of parameters are not necessarily convergent toward their true values, i.e., z may converge to some nonzero constants.

Typical SMC strategies for MIMO systems include the following.

1) Equivalent control-based SMC. This is a control of the type

$$u = u_{\mathrm{eq}} + u_{\mathrm{s}} \tag{8}$$

where u_{s} is a switching control component which may have two types of switching
a) $u_{\mathrm{s}_i} = -\alpha_i(x)\mathrm{sgn}(s_i)$ for $\alpha_i(x) > 0$;
b) $u_{\mathrm{s}_i} = -\beta(s_i/\|s_i\|)$, $\beta > 0$
and there are other variants.

2) Bang-bang-type SMC: This is a control of direct switching type

$$u_{\mathrm{s}_i} = -M\mathrm{sgn}(s_i)$$

where $M > 0$ should be large enough to suppress all bounded uncertainties and unstructured systems dynamics. Design of such control relies on the Fillipov method, by which, a sufficient large local attraction region is created to suck in all system trajectories.

3) Enforce

$$\dot{s} = -R\mathrm{sgn}s - K\sigma(s) \tag{9}$$

to realize finite time reachability [9], where $R > 0$ and $K > 0$ are diagonal matrices and $s_i\sigma(s_i) < 0$.

4) Embedding adaptive estimation and learning of parameters or uncertainties in 1), 2), and 3), which largely follows the methodologies of adaptive control, e.g., [10], [11] and iterative learning control mechanism, e.g., [12]. In recent years, there has been some new mechanisms developed for improving SMC performance, such as the terminal sliding modes [13] in which fractional power is introduced to improve the SMC convergence speed and precision, the second-order SMC [14], [15] which has the advantage of reducing chattering as well as not requiring derivative information for the control, and the higher order SMC [16] which makes use of the sequential integration to smooth chattering.

Perhaps it is worth mentioning that so far, the SMC of linear time invariant systems has been the dominating system type studied in the literature. In this context, the system (1) becomes

$$\dot{x} = Ax + Bu + \xi(x,t) \tag{10}$$

and $s(x)$ is a set of switching hyperplanes defined as $s(x) = Cx = 0$, where C is a constant $m \times n$ matrix to be determined. For the original control u (assume the matrix (CB) is nonsingular), $u_{\mathrm{eq}} = -(CB)^{-1}CAx$ such that under the control the dynamics in the sliding mode becomes

$$\dot{x} = \left[I - B(CB)^{-1}C\right]Ax = A_{\mathrm{eq}}x. \tag{11}$$

Notice that during the sliding, the system dynamics becomes $n - m$ dimensions due to the constraint $Cx = 0$ confined to $\dot{s}(x) = 0$. The matrix A_{eq} is actually a *projection operator* along the range space of B onto the null space of C [17], i.e., $A_{\mathrm{eq}}B = 0$, $A_{\mathrm{eq}}x = x\ \forall x \in \mathbb{R}^n$ subject to $Cx = 0$, where C can be designed such that the $n - m$ eigenvalues of A_{eq} are allocated in the left-hand side of the complex plane and remaining m eigenvalues remain zero.

C. Key Issues in SMC Theory and Applications

There are several key issues that are commonly seen as obstacles affecting the wide spread applications of SMC, which have prompted the extensive use of SC and other technologies in SMC systems in recent years. Some of these are listed in the following.

1) Chattering: As has been previously mentioned, SMC is a special class of VSSs, in which the sliding modes are induced by disruptive control forces. As demonstrated in the previous sections, despite the advantages of simplicity and robustness, SMC generally suffers from the well-known problem, namely, *chattering*, which is a motion oscillating around the predefined switching manifold(s). Two causes are commonly conceived [18].

1) The presence of parasitic dynamics in series with the control systems causes a small-amplitude high-frequency oscillation. These parasitic dynamics represent the fast actuator and sensor dynamics which are normally neglected during the control design.
2) The switching nonidealities can cause high-frequency oscillations. These may include small time delays due to sampling [e.g., zero-order hold (ZOH)], and/or execution time required for calculation of control, and more recently, transmission delays in networked control systems.

Various methods have been proposed to "soften" the chattering. Examples are as follow.

1) The boundary layer control in which the sign function is replaced by $\mathrm{sgn}_b(s_i) = \mathrm{sgn}s_i$, if $|s_i| > \epsilon_i$, $\epsilon_i > 0$, and $\mathrm{sgn}_b(s_i) = \rho(s_i)$ if $|s_i| \leq \epsilon_i$ where ϵ_i is small and $\rho(s_i)$ is a function of s_i which could be a constant or a function of states as well [19].
2) The continuous approximation method in which the sign function is replaced by a continuous approximation $\mathrm{sgn}_a(s_i) = (s_i/(|s_i| + \epsilon))$ where ϵ is a small positive number [20]. This, in fact, gives rise to a high-gain control when the states are in the close neighborhood of the switching manifold.

2) Matched and Unmatched Uncertainties: It is known that the celebrated invariance property of SMC lies in its insensitivity to matched uncertainties when in the sliding mode [8]. However, if the matching condition is not satisfied, the sliding mode (motion) is dependent on the uncertainties ξ which is not desirable because the condition for the well-known robustness of SMC does not hold. Research efforts in this aspect have been focused on restricting the influence of the uncertainties ξ within a desired bound.

3) Unmodeled Dynamics: Mathematically, it is impossible to model a practical system perfectly. There always exists some unmodeled dynamics. The situation may be worsened if the unmodeled dynamics contains high-frequency oscillatory dynamics which may be excited by the high-frequency control switching of SMC. There are several aspects of dealing with this unmodeled dynamics.

1) The unmodeled dynamics can refer to the presence of parasitic dynamics in series with the control system as shown above.
2) Due to the complexity of practical systems, the unmodeled dynamics which may have different dimensions and characteristics cannot be modeled at all. In this case, the unmodeled dynamics can be commonly lumped with uncertainties and a sufficiently large switching control has to be imposed. Certainly, the more the knowledge of the uncertainties is, the less crude will be the SMC force.

More broadly, in the complex systems environment, the object to be controlled is difficult to model without a significant increase in the dimensions of the problem space. Often, it is more feasible to study the component subsystems linked through aggregation. The challenge is how to design an effective SMC without knowing the dynamics of the entire system (apart from the aggregative links and local models). This research topic has been very popular in recent years in the intelligent control community using, for example, NNs, FL, and PR. This is one of the focus areas of this survey.

4) Interconnections Between Chattering, Matched and Unmatched Uncertainties, and Unmodeled Dynamics: The issues of chattering, matched and unmatched uncertainties, and unmodeled dynamics are interconnected. Take the SMC system under the equivalent control (4) with $u_{\mathrm{s}} = -\alpha \mathrm{sgn}(s)$. Examples are as follow.

1) If we know the uncertainties ξ completely, the switching magnitude α does not need to be large to ensure the switching manifold $s = 0$ is reached in finite time. Of course, if one is concerned with the reaching speed to the switching manifold, one has to increase the control magnitude α to shorten the reaching time.
2) If we do not know the uncertainties ξ, provided they are bounded, we can always find a large enough α to overcome the "adverse" influence of ξ. This, in turn, results in severe chattering if the switching device is not ideal. This applies to the unmodeled dynamics as well if we have no knowledge of it at all.
3) If there is a way that we can at least partially know the uncertainties or the unmodeled dynamics, by alternative learning mechanisms such as NNs and fuzzy systems, the partial knowledge can help reduce the requirement for large switching magnitude α, hence reducing the tendency of larger chattering. This is the core reason behind many SC-based solutions.

5) Sliding, Nonsliding, and Zeno Motions: There are three possible motions that can be resulted due to switching-type controls: sliding, nonsliding, and zeno motion, each of which has their own characteristics. Sliding motion refers to the motion that a subsystem becomes an attractor sucking in all system states. Nonsliding motion refers to the motion where the switching manifolds representing a subsystem that are not attractive and switchings that occur correspond to the crossings of the manifolds [21]. The third type is the so-called zeno motion, which refers to infinite switching resulting in finite reaching time [22].

In many cases, particularly in practical control systems, because of the physical restrictions, a theoretically global SMC strategy does not exist, i.e., they are always confined within a limited domain in the state space because of the physical limits, no matter how large it is. Typically, assume the upper and the lower bounds of control magnitudes are denoted as $\underline{u}$ and $\bar{u}$, then the attraction region of the sliding motion is confined within $\underline{u} < u_{\mathrm{eq}} < \bar{u}$ [23]. It must be emphasized that, to be able to enjoy the invariant properties of SMC, finite time reachability of the switching manifolds must be realized. In many research papers, the reachability condition is loosely defined as only being $\dot{V} < 0$ which would result in asymptotical stability, i.e., the switching manifolds are never reached in finite time, rather in infinite time. In this case, the system does not exhibit the invariant properties rigorously.

Often, in the open literature, the above motions are studied alone, and the individual school of thought for each type of motion is stringently followed. However, in reality, the induced motion could be a mixed one, i.e., in one region of the state space, the system exhibits a sliding motion, while in another region, it exhibits a nonsliding motion.

III. SC Technologies

In this section, we will give some brief introduction to the key SC techniques which are commonly used in dynamical systems and control. Components of SC technologies are NNs, FL, and PR which subsume belief networks, evolutionary computation (EC), chaos theory, and parts of learning theory [6].

A. NNs

An NN is a computational structure that is inspired by observed processes in natural networks of biological neurons in the brain [24]. It consists of a mass of neurons, each of which is a simple computational unit, yet the intersections between the neurons emulate the enormous learning capability of the brain. This learning is done by adjusting the so-called weights that represent the interconnection strength of neurons, according to certain learning algorithms. In general, the learning can be supervised or unsupervised. In a supervised learning algorithm, learning is guided by specifying, for each training input pattern, the class to which the pattern is supposed to belong. The adjustments of weights are done to minimize the difference between the desired and actual outputs incrementally. A common structure is the feedforward NNs (FNNs) [24] where the information flow in the network is directional.

Supervised learning in FNNs appears to be the most popular research area in dynamical systems and control applications. Generally, it can be formulated as follows. Given the inputs, weights, desired outputs, and actual outputs of an NN as $x \in R^n$, $w \in R^l$, $y_d \in R^m$, and $y \in R^m$, respectively, the target, for learning purposes, is to minimize the performance index, which can be generically written as

$$J = \frac{1}{\tau} \int_{t-\tau}^{t} \frac{1}{2} \|y(\theta) - y_d(\theta)\|^2 \tag{12}$$

where τ is the length of time window and $\|.\|$ represents the Euclidean norm.

Most supervised learning [backpropagation (BP)] algorithms for NNs can be considered as finding zeros of $\partial J/\partial w$ which correspond to their local as well as global minima. The search performance of this class of learning algorithms somehow relies on initial weights and, frequently, it traps into local minima. As shown in [25], all FNN learning can be generally regarded as finding minimum for the Lyapunov function

$$V = J + \frac{1}{2}\mu \left\| \frac{\partial J}{\partial w} \right\|^2 \tag{13}$$

where the parameter μ determines the relative importance of each term in the function.

One particular FNN which offers a much faster way of learning is the so-called radial basis function-based NN (RBFNN) [26]. In this setting, the network structure is much simpler, i.e., $y_i(t) = \sum_{j=1}^{m} w_j^i \phi_j(x - \mu_j, \theta_j)$, where $\phi(x - \mu_j, \theta_j)$ is a Gaussian function. The learning of the weights becomes much simpler as well, using the conventional FNN learning paradigm. Even though, extensive computation is still required and efficient algorithms can be derived [27].

Another class of NNs is the so-called recurrent NNs (RNNs) which has a feedback mechanism [24], similar to dynamic feedback systems. This class of NNs has also been popular in the area of dynamical systems.

It should be pointed out that while NNs offer a *model-free* approach for system representation, which appears attractive, one should not underestimate the efforts of getting the solutions through NNs. This can well be explained by the *no free lunch* theorem [28] which stipulates that a general-purpose universal optimization strategy is theoretically impossible, and the only way one strategy can outperform another is if it is specialized to the specific problem under consideration. This means that the more you know about the problem you deal with, the faster you get the solution.

B. Fuzzy Systems

A fuzzy system is any system whose variables (or, at least, some of them) range over states that are fuzzy sets based on FL, which is a form of multivalued logic derived from fuzzy set theory to deal with reasoning that is vague rather than precise. In FL, the degree of truth of a statement can range between zero and one.

For each variable, the fuzzy sets are defined on some relevant universe of discourses, which are often an interval of real numbers. In this special but important case, the fuzzy sets are fuzzy numbers, and the associated variables are linguistic variables. Representing states of variables by fuzzy sets is a way of quantifying the variables. Fuzzy systems provide an alternative representation framework to express problems that cannot easily be described using deterministic and probabilistic mathematical models.

In the domain of control systems, the output of a fuzzy system can be computed by a mechanism of Takagi–Sugeno–Kang (TSK)-type IF-THEN rules [29], [30], which can be described as follows:

$$R^i: \quad \text{IF} \quad z_1 \text{ is } F_1^i \text{ AND} \cdots z_n \text{ is } F_n^i \quad \text{THEN} \quad y^i = c^i \tag{14}$$

for $i = 1, 2, \ldots, m$, where R^i represents the ith fuzzy inference rule, m is the number of inference rules, $F_i (i = 1, \ldots, n)$ are the fuzzy sets, $z(t)$ is the system state, u is the system input, and y_i is the system output which could be a function c^i. Denote $w^i(z(t))$ as the normalized fuzzy membership function of the inferred fuzzy set F^i where $F^i = \bigcap_{j=1}^{n} F_j^i$, the output of the fuzzy system is formulated as $y = (\sum_{i=1}^{m} w^i y^i)/(\sum_{i=1}^{m} w^i)$. Another well-known fuzzy control method is the Mamdani method [31] which makes use of much intuitive nature of human expert control. However, the TSK method is more suitable for model-based fuzzy control systems.

In the domain of control systems, dynamical systems are commonly dealt with. Hence, in this case, the output function $y^i = c^i$ can be replaced with a dynamical equation, which can be viewed as a local model.

C. PR

PR refers to EC, chaos theory, belief networks, and parts of learning theory [6]. Below are some brief overviews of each area.

1) EC: Among PR technologies, EC techniques have been a most widely used technology for optimization in SMC. The EC paradigm attempts to mimic the evolution processes observed in nature and utilize them for solving a wide range of optimization problems. EC Technologies include genetic algorithms (GAs), genetic programming, evolutionary algorithms, and strategies. In general, EC performs directed random searches using mutation and/or crossover operations through evolving populations of solutions with the aim of finding the best solutions (the fittest survives). The criterion which is expressed in terms of an objective function, is usually referred to as a fitness function [32].

2) Chaos Theory: One emerging SC technology is the chaos theory which can be used to explain complex behaviors from rather simple dynamical systems. In mathematics, chaos theory describes the behavior of certain dynamical systems that may exhibit dynamics that are highly sensitive to initial conditions. As a result of this sensitivity, which manifests itself as an exponential growth of perturbations in the initial conditions, the behavior of chaotic systems appears to be complex, irregular and randomlike. Chaos has been found in many different physical systems, such as chemical reactors, fluid dynamics, forced oscillators, feedback control devices, and laser systems [33].

3) Other Technologies: There are other SC methodologies that are presently seldom used with SMC. Belief networks are one such example, partly due to the industrial domain in which most SMC applications are seen. In such an environment, human reasoning is presently resorted to for decision making at higher levels. However, belief networks may become useful in the future, when the use of SMC in complex systems control involving human activities is considered. The same reasoning applies to learning theory. Complex networks theory is another technology which can be used to explain emerging behaviors in highly connected networks of individual subsystems [34]. However, these theories are unlikely to be the major tools for SMC designs in the near future.

D. Integration of SC Methodologies

The aforementioned SC paradigms offer different advantages. The integration of these paradigms would give rise to powerful tools for solving difficult practical problems. It should be noted that NNs and EC are about a process that enables learning and optimization while the FL systems are a representation tool. Emerging technologies such as swarm optimization, chaos theory, and complex network theory also provide alternative tools for optimization and for explaining emerging behaviors which are predicted to be widely used in the future.

IV. SMC With SC

There have been extensive research done in using SC technologies for SMC systems. An earlier survey [35] already outlined some of the major developments. Since then, significant research has been progressed and this paper aims to depict the state of the art of SMC with SC. We will use the same categories as in Section III to summarize the developments.

As has been stated before, the integration of SMC and SC has two dimensions. One is the application of SC technologies in SMC to make it "smarter" and the other using SMC to enhance SC capabilities [36]–[40]. This survey will be confined within the scope of the former, i.e., the application of SC technologies in SMC.

A. SMC With NNs

As has been outlined before, there are three essential interconnected aspects that affects usability and applications of SMC: chattering, matched and unmatched uncertainties, and unmodeled dynamics. NNs offer a "model-free" mechanism to learn from examples the underlying dynamics. Conventional NNs learn by employing BP type of learning as stipulated in [24] and [25]. This learning process is usually very time consuming.

Unlike conventional learning of constant parameters or functions, learning speed is very important in dynamical environments. Reducing learning errors rather than obtaining exact parameter values is of paramount importance. In control theory, the application of NNs is done in the context of dynamical learning focusing on achieving stability and fast convergence, rather than reducing learning errors. The convergence of learning parameters to their true values becomes secondary, although, under certain conditions, for example, in adaptive learning and control, when the persistence excitation condition is satisfied, such convergence is guaranteed [19].

One of the early attempts in using NNs in SMC [41] aims to estimate the equivalent control. An online NN estimator is constructed, and the NN is trained using the alternative function $V = 0.5(\dot{s} + ds)^2$ instead of (6). The same approach is also contemplated in [42]. In another work [43], two NNs are used to approximate the equivalent control and the correction control, respectively. NNs in control systems adopt the same topological structures but the learning is done through real-time estimation.

In terms of learning mechanisms, while the conventional BP-based NN learning paradigm is usable, there is another trend of using the control design techniques to derive the learning algorithms with convergence to the true parameter values being a secondary requirement. Instead of using the conventional NN performance index (13) for learning, a performance index of adaptive control type (7) is used, where z is defined as functions that enable the convergence of NN weights to deliver the convergence $V \to 0$. In the literature, there are several research works using similar ideas. For example, in [44], a dynamical NN approach is used to estimate the unmodeled dynamics to reduce the requirement of large switching control magnitude using the Lyapunov function (7), the stability as well as modeling of the uncertainties are guaranteed for an electronic throttle. In [45], the switching manifolds are used as performance criteria and adaptive algorithms are derived for learning and control.

In [46], an NN estimator is used to estimate the modeling errors to compensate the SMC to reduce the tracking errors in discrete nonlinear systems. In [47], an NN is used to estimate the lumped unmatched uncertainties with time delayed states. Adaptive algorithms are derived for estimating parameters and guarantee stability. It is indicated that the normal requirement of bounded uncertainties is not required in the control scheme. In [48], two NNs are used to estimate the equivalent control and the magnitude of the switching control component using the conventional gradient learning algorithm. This approach is proved to be effective in reducing chattering and practically implemented on a seasaw system.

For better learning efficiency, the RBFNNs have become a popular option because, with such networks, less number of parameters are to be learned, hence the convergence speed is enhanced. For example, in [49], an RBFNN is used as a model-free approximator for an adaptive SMC which can deal with online learning of uncertainties.

RNNs have also been considered to be a good candidate for incorporation into SMC structures because of its feedback mechanism which enhances convergence. For example, in [50], a special RNN is constructed to estimate the equivalent control in order to reduce chattering, together with a special learning algorithm for updating the parameters to be learnt. In [51], an RNN is used to estimate the uncertainty to reduce the impact of the uncertainty bound on SMC of antilock breaking systems, and a compensator to compensate the difference between the true uncertainty and the estimated uncertainty. The RNN has proved to be superior to the FNN mechanism. In [52], a self-recurrent wavelet NN is used to estimate the unmodeled uncertainties (and the model), as well as the external disturbances, for controlling nonholonomic wheeled mobile robots.

The development is based on the Lyapunov function (7). In [53], an RNN is used to facilitate the adaptation process of a robust SMC for robotic manipulators.

B. SMC With FL

The strength of fuzzy systems based on FL has been taken advantage of in SMC design due to its simple representation, underpinned by the heuristic nature of human reasoning. The rationale central to the use of fuzzy systems with SMC is similar to those for NNs, i.e., to alleviate the problems associated with chattering, matched and unmatched uncertainties, and unmodeled dynamics. The key advantages include introducing heuristics underpinned by human expert experience to reduce (or soften) chattering, to model the unmodeled uncertainties based on partial knowledge of experts through their years of experience, and to control complex systems which are modeled through aggregated linear models via FL. This section will outline major developments in this respect.

1) Dealing With Chattering: One of the early works in this respect uses FL in a low-pass filter to "smooth" SMC signals, which would reduce chattering [54]. This is done by forming a specifically constructed fuzzy rule table.

Another approach to dealing with chattering is to fuzzify SMC. Fuzzifying the sliding-mode concept and the reaching phase to the sliding manifold involves replacing the otherwise well-defined mathematical derivations with heuristic rules underpinned by FL. There have been extensive works in this area, for example, [55]–[58].

One of the earliest works in which the fuzzy concepts are introduced is [59]. In this paper, instead of using the well-defined SMC functions, the control action is derived from an FL-based control table, compounded by fuzzy membership functions. In such a way, the landing on the switching manifold and overshooting it is softened and this eases the chattering. In [60], the softening FL idea is introduced in tuning the switching magnitude according to the distance from the state to the switching manifold. In [61], the same idea is utilized in tuning the control force according to the actual position to the switching manifold and the velocity toward it.

The main idea behind the use of SMC with FL is to use human control intuition and logic to enhance SMC performance. For example, one simple and direct way to design a fuzzified SMC is by using the actual position to the switching manifold and the velocity toward it as two inputs and to derive rules as to what control action is required for the state to reach the switching manifold. Consider the case when the following standardized fuzzy values are used, i.e., NB = "negative big," NS = "negative small," ZO = "zero," PB = "positive big," and PS = "positive small" are used. If the state of the trajectory to the switching manifold is NB and the velocity toward the switching manifold is NS, an intuitive control action should be PB since only a drastic action can reverse the speedy trend of the trajectory leaving the switching manifold. However, if the state of the trajectory to the switching manifold is PB and the velocity toward the switching manifold is NB, meaning that the system dynamics has a useful tendency toward the switching manifold, one may choose NS or NB as the control output to further accelerate the convergence. There are many choices, depending upon what control performance is required. Certainly, the control choices are not unique. The key to fuzzified SMC lies in the selection of membership functions associated with the fuzzy values.

The aforementioned example serves as a typical case for fuzzified SMC and similar ideas have been used in a number of papers. For example, in [62], the fuzzified SMC is used for controlling a cart-pole system. In [63], a decoupled fuzzy SMC is proposed using the fuzzified SMC concept. In the literature, various other fuzzified SMC applications can be seen. For example, in [64], a fuzzy SMC with output feedback is designed. In [65], an interesting hybrid fuzzy rule is proposed to adjust the control gain. FL is also used in tuning sliding-mode controllers [66]. In a recent work [67], a robust fuzzy sliding-mode controller is proposed for active suspension of a nonlinear half-car model in which the ratio of the derivative of the error to the error is used as the input to the controller.

2) Dealing With Uncertainties: Uncertainties are another cause of chattering. Efforts to reduce chattering are directed to ensure that information about the uncertainties and the unmodeled dynamics be obtained as much as possible in order to reduce the requirements of high enough (crude) switching gains to suppress them and to secure the existence of sliding modes and hence the robustness. The estimation of uncertainties using fuzzy systems has been quite a popular approach. In this section, we also discuss uncertainties associated with unmodeled dynamics, i.e., the uncertainties also include lumped unmodeled dynamics, which may refer to the presence of parasitic dynamics in series of the control systems that causes a small amplitude high-frequency oscillations. These parasitic dynamics are normally neglected during the control design. It may also refer to other residual dynamics that cannot be modeled. However, the latter is difficult to handle due to the uncontrollable nature of the unmodeled dynamics.

A fuzzy system architecture is proposed in [68] to adaptively model the nonlinearities and the uncertainties. In [69] and [70], schemes are proposed to estimate the unknown functions in order to reduce the magnitudes of the switching controls. Tao *et al.* [71] propose an adaptive fuzzy SMC to learn the equivalent control and two switching controls without *a priori* knowledge of the bounds of the uncertainties. In [72], an adaptive fuzzy SMC is proposed in which the fuzzy systems are used to estimate the equivalent control and the switching control.

3) Dealing With Complex Systems: More broadly, in the complex systems environments, the object to be controlled is difficult to model without a significant increase in the dimension of the problem space. The feasible solutions in such cases are often to study the component subsystems, networked through a topological structure. The challenge is how to design an effective SMC without knowing the dynamics of the entire system (apart from the topological links). This direction of research has become very popular in recent years in the intelligent control community using, for example, NNs, FL, and EC technologies.

For complex dynamical systems, it is possible to linearize them around some given operating points such that the well-developed linear control theory can be applied in the local region. Such a treatment is quite common in practice. However, there may exist a number of operating points in complex nonlinear systems that should be considered during control. How to aggregate the locally linearized models into a global model to represent the complex system is not an easy task. One effective approach is the FL approach, by which the set of linearized mathematical models is aggregated into a global model that is equivalent to the complex system. Various fuzzy models and their controls have been discussed, for example, Sugeno and Kang [73], Takagi and Sugeno [29], Tanaka and Sugeno [74], and Feng *et al.* [75]. Such a formulation can be easily accommodated by replacing $y^i = c^i$ by a dynamic linear system [75]

$$\dot{x}(t) = A_i x(t) + B_i u(t) \quad y_i(t) = D_i x(t) + E_i$$

where the matrices A_i, B_i, D_i are of appropriate dimensions and E_i is a constant vector. Using the standard fuzzy inference approach, the global fuzzy state space model can be obtained as

$$\dot{x}(t) = Ax(t) + Bu(t) \quad y = Dx(t) + E \tag{15}$$

where $A = \sum_{i=1}^{m} \mu_i A_i$, $B = \sum_{i=1}^{m} \mu_i B_i$, $D = \sum_{i=1}^{m} \mu_i D_i$, $E = \sum_{i=1}^{m} \mu_i E_i$. While the global fuzzy model can approximate the complex system, the aggregated SMC cannot guarantee the global stability. This is because the globally asymptotical stability is not guaranteed in the overlapping regions of fuzzy sets where several subsystems are activated at the same time, unless it satisfies a sufficient condition [76]. In [77], the dynamics of (9) is used to enforce the reachability of the sliding modes for MIMO nonlinear systems represented in a similar form, as shown in above. A fuzzy adaptive control scheme is then developed to deal with the model following control problem. In another work [78], robust adaptive fuzzy SMC is proposed for nonlinear systems approximated by the Takagi–Sugeno fuzzy model. Recently, Liang *et al.* [79] use the Takagi–Sugeno fuzzy model for robot control and report rapid response and robustness, and Park *et al.* [80] develop an adaptive fuzzy SMC for an uncertain crane system incorporating a fuzzy observer for uncertainties.

C. SMC With PR

One PR technology commonly used in SMC is the EC for optimization. Another area is the application of chaos theory in SMC. We shall discuss their applications in SMC in the following.

1) SMC With EC: The optimization of the control parameters and the model parameters are vitally important in reducing chattering and improving robustness. In this respect, various optimization techniques can be used [24] such as the gradient-based search, Levenberg–Marquart algorithm, etc.; however, a significant amount of information about the tendency of search parameters toward optima (i.e., the derivative information) is required. The complexity of the problem and the sheer number of parameters make the application of the conventional search methods rather hard. The key process of using EC is to treat the set of parameters (often a large number) as the attributes of an individual in a population and generate enough number of individuals randomly to ensure a rich diversity of "genes" in the population, through bioinspired operations such as mutation and/or crossover.

The advantages previously mentioned has motivated various researchers to use EC as an alternative to find "optimal" solutions for SMC. Lin *et al.* [81] use a GA to reach an online estimation of the magnitude of switching control to reduce chattering. In [82], a GA is used to optimize the matrix selection to obtain a better reaching phase. In [83], a GA is used to optimize the matrix selection to obtain better membership functions for smoother SMC. Some practical guidance for the use of GA in SMC design can be found in [84]. Lin and Chen [85] propose two levels of optimization using GA to optimize the fuzzy SMC structure. In [86], a chaos-based GA is developed to tune the terminal SMC for a two-link flexible robotic manipulator. Tseng *et al.* [87] derive a GA-based SMC for optimal speed control of synchronous reluctance motors where the GA is used to search the uncertainty parameters.

2) SMC With Chaos Theory: It is difficult to envisage how chaos theory can be incorporated into SMC although it can assist to explore the inherent nonlinear behaviors to help design smarter controllers. This area is largely not well researched. Chaotic behaviors may occur in the digitization/discretization of SMC. For example, for a relatively simple Euler discretization, quite complex discretization behaviors are observed [88]. More complex behaviors are discovered in ZOH discretization of SMC [89]. Such complex behaviors are also found with delta-modulation feedback in discretization of SMC [90]. Nevertheless, this is an emerging area of research in SMC.

D. SMC With Integrated NN, FL, and EC

It is recognized that NNs and EC are processes that enable learning and optimization while the fuzzy systems are a representation tool [35]. For complex systems which are difficult to model and represent in an analytic form, it is certainly advantageous to use fuzzy systems as a paradigm to represent the complex systems as an aggregated set of simpler models, just like the role of local linearization plays in nonlinear function approximation by piecing together locally linearized models. NNs on the other hand can be used as a learning mechanism to learn the dynamics while the EC approaches can be used to optimize the representation and learning. Such ideas have been used quite extensively in dynamic systems and control areas. Research works in this area include Da [91] in which two fuzzy NNs are used to learn the control as well as identify the uncertainties to eliminate chattering in distributed control systems. The conventional BP-based learning mechanism is employed in this paper for learning. In [92], a robust fuzzy SMC is proposed for controlling motion control systems. The learning mechanism is used to approximate the equivalent control. In [93], a fuzzy NN is used in a tracking control system for a permanent magnet synchronous servomotor drive, which contains a supervisory control and an FNN SMC to improve the robustness. The control signal is learned through a BP search algorithm. In [94], a novel fuzzy NN approach based on the terminal sliding modes is proposed to estimate the uncertainties for control applications and notable improvements in precision are demonstrated.

V. Interplay of SMC and SC

In the aforementioned sections, the state of the art in SMC with SC has been examined. In what follows, some views are shared with the readers about the interplay of SMC and SC and their potential for future research.

A. New SMC Concepts

Although SMC has been studied for over half a century, the research in this area has continued to be very active. In recent years, new SMC mechanisms such as second-order SMC [14], terminal SMC [13], and higher order SMC [16] have been introduced which show promising dynamical properties such as finite time convergence and chattering alleviation. The introduction of useful nonlinearities such as nonsmoothness has been shown to offer attractive properties in SMC designs. While the new SMC concepts provide promising dynamic properties that should be taken advantage of, they also result in mathematical difficulties due to the nonsmoothness and discontinuities. It is not known whether these new SMC concepts will work well with SC technologies, although some preliminary work does show some promising aspects [94]. Further work in this area is needed.

B. SC With SMC

One important aspect of the fusion of SC methodologies and SMC is the application of SMC in SC technologies. Due to the page limit and the focus of this paper, this aspect has not been discussed extensively. However, this direction of research has also been very active, some examples being the use of SMC to unify learning algorithms for FNN [25], to improve NN learning efficiency [36], [37], [39] and to enhance optimization [95], [96]. It is believed that future research in this area will also be very promising because of the demands for efficiency and robustness in intelligent learning systems, which can benefit from the robustness and fast responsiveness of SMC.

C. SMC in Complex Networked Systems

In modern industrial systems with a large number of distributed sensors and actuators, information is required to be transmitted and operations be coordinated through a communication network or shared medium [97]. The control systems wherein the control loops are closed via a communication network. The advanced communication technology has provided a low-cost solution for cost-effective distributed information processing. However, the inherence problems of network-induced delay and data packet dropout pose new challenges to the implementation of SMC in networked environments. This area of research has just started recently. Examining the research issues of SMC in the networked environments thoroughly is very important for the successful applications of SMC systems. Typical problems such as chattering, uncertainties, and unmodeled dynamics would get worsened because of the possible time varying delays and data packet dropouts. Certainly, the entire SMC theory needs to be revisited specifically for the networked environments to safeguard their successful applications. Furthermore, it is also not known whether the incorporation of SC technologies in SMC in networked environments would be advantageous. Solving these problems will also have a significant impact on future applications of SMC.

VI. Conclusion

In this paper, the state of the art of SMC with SC has been examined and their future aspects have been outlined. Although SMC has been studied for more than 50 years, new technological challenges in the 21st century require new breakthroughs in SMC theory and applications. This is indeed an exciting research field to get into.

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