# Dynamical Modeling of Two-Wheeled Cart: Cooperating Manipulator Approach

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***Abstract*— Two-wheeled cart falls under kinematically deficient manipulator when considered from cooperating manipulator approach. Kinematically deficient manipulators are those that have fewer degrees of freedom than necessary to achieve any admissible configuration in their operational space. When multiple manipulators, some or all of which are kinematically deficient, cooperate to perform a common task, the constrained forces at the contact points cannot be solved directly due to rank deficiency of the jacobian. This paper addresses this challenge associated with the computation of constrained forces at the contact points by introducing a novel approach called "pseudo joint." Forward dynamical model utilizing pseudo joint has been driven for cooperating kinematically deficient manipulators. Dynamics of two-wheeled cart has been given to demonstrate the methodology.**

## I. Introduction

Kinematically deficient manipulators are those that have fewer degrees of freedom (DOF) than necessary to achieve any admissible configuration in their operational space. Many industrial applications do not require the full kinematic capability to move and rotate the tip point of the manipulator in any direction. Usually, the desired trajectory lies in a subset of this six dimensional operational space. Unless kinematic redundancy is needed for both task space and joint space controls such as obstacle avoidance or joint limit avoidance problems, kinematically deficient manipulators gain superiority over more DOF manipulators in terms of cost, manufacturing, and compactness. In addition, cooperating manipulators bring unprecedented advantage over serial manipulators in terms of precision, load balancing, high payload capacity, etc. Therefore, certain applications require to utilize multiple manipulators that cooperate to perform a common task and are kinematically deficient.

Although constrained manipulators and kinematically redundant manipulators have been studied extensively, such as the work by Bruyninckx and Khatib [3], kinematically deficient manipulators have not attracted enough attention from the scientific community. Abdel-Malek et al. [1] studied the workspace issues of kinematically deficient manipulators. Dynamics of two-finger grippers as kinematically deficient manipulators was studied by Prattichizzo and Bicchi [6]. Teleoperated surgical robots were considered in both kinematically redundant and kinematically deficient cases by Funda et al. [5].

The class of cooperating manipulators account for a significant importance in the robotics. On the other hand, they bring a challenge in the computation of dynamical model where the contact forces need to be calculated. Yoshikawa and Zheng [9], and Deo and Walker [4] are among the researchers who studied this problem. Anderson and Critchley [2] proposed a high performance algorithm for the mentioned class of systems.

This paper is aimed at addressing the numerical problems associated with contact force calculations by introducing a new concept called "pseudo joint." The next section starts from multiple serial manipulators without forming closed loops and then explains the dynamical model of cooperating manipulators. In section 3, the challenges associated with kinematically deficient manipulators are considered. Pseudo joint methodology is introduced. Finally, the conclusion is given in section 4.

## II. Dynamics of Cooperating Manipulators

### A. Nomenclature

The algorithm presented here utilizes a basis-free vectorial representation. The body coordinate frame is denoted by $O$. Right subscript of a variable indicates the associated link. In the case of cooperating manipulators each manipulator is represented by a number. Left superscript of a variable indicates this number. For example, ${}^2O_3$ is the body frame on the third link of the second manipulator. Right subscript indicates the size. For example ${}_3I$ is the $3 \times 3$ identity matrix, ${}_50$ is the $5 \times 5$ zero matrix. ${}^ih_k$ is a unit vector parallel to the axis of rotation of the joint at the $k$th link of the $i$th manipulator. This definition is done under the assumption, without loss of generality, that each link has one DOF joint. In the presence of multiple DOF joints this assumption can easily be removed by making ${}^ih_k$ include one unit vector per column. ${}^iH_k$ and ${}^i\theta_k$ are the spatial axis of motion and the joint angle for the aforementioned joint, respectively. ${}^i\ell_{k,k+1}$ is a vector from ${}^iO_k$ to ${}^iO_{k+1}$. The linear and angular velocity vectors with respect to the inertial frame which is chosen as the base frame, $O_0$, are ${}^iv_k$ and ${}^i\omega_k$. Spatial velocity and the spatial acceleration of the link are denoted as ${}^iV_k$ and ${}^i\alpha_k$ and they are defined as $\left( \begin{array}{cc} {}^i\omega_k^T & {}^iv_k^T \end{array} \right)^T$ and $\left( \begin{array}{cc} {}^i\dot{\omega}_k^T & {}^i\dot{v}_k^T \end{array} \right)^T$,respectively, where $^T$ is the transpose operator. ${}^im_k$ is the mass and ${}^i\mathcal{I}_k$ is the inertia tensor at point ${}^iO_k$.${}^i\ell_{k,c}$ is the vector from ${}^iO_k$ to the links center of mass. Link forces and torques acting at ${}^iO_k$ are denoted as ${}^if_k$ and ${}^i\tau_k$. Link spatial force vector ${}^iF_k$ is defined as $\left( \begin{array}{cc} {}^i\tau_k^T & {}^if_k^T \end{array} \right)^T$. Finally, skew symmetric linear operator of vector cross product is denoted by *hat*, i.e, $\widehat{p} = \vec{p}\times$.

### B. Dynamics of multi-manipulators forming a tree structure

In this section equation of motion will be derived for multiple serial manipulators [8]. Angular and linear link velocities of the $i$th manipulator with respect to its base frame, propagate from link $k-1$ to link $k$ as follows:

$$ {}^iV_k = {}^i\phi_{k,k-1}{}^iV_{k-1} + {}^iH_k{}^i\dot{\theta}_k $$

where

$$ {}^i\phi_{k,k-1} = \begin{bmatrix} {}_3I & {}_30 \\ -{}^i\widehat{\ell}_{k-1,k} & {}_3I \end{bmatrix} $$

${}^i\phi_{k,k-1}$ is a linear operator that translates velocities from $O_{k-1}$ to $O_k$ and called the “propagation operator.” Velocity vector, propagation operator and joint axes matrix are written for the $i$th manipulator as:

$$ {}^iV = \begin{bmatrix} {}^iV_1 \\ {}^iV_2 \\ \vdots \\ {}^iV_n \end{bmatrix} \quad {}^i\phi = \begin{bmatrix} {}_6I & {}_60 & \cdots & {}_60 \\ {}^i\phi_{2,1} & {}_6I & \cdots & {}_60 \\ \vdots & \vdots & \ddots & \vdots \\ {}^i\phi_{n,1} & {}^i\phi_{n,2} & \cdots & {}_6I \end{bmatrix} $$

$$ {}^iH = diag({}^iH_1, {}^iH_2 \ldots {}^iH_n) $$

where $n$ is the number of DOF of the $i$th manipulator. If there are $p$ many manipulators in the system, quantities above can be staked up as $V = [{}^1V^T \cdots {}^pV^T]^T$, $\phi = diag({}^1\phi \cdots {}^p\phi)$, $H = diag({}^1H \cdots {}^pH)$. Based on these definitions, the link velocities are obtained as follows:

$$ V = \phi H\dot{\theta} \tag{1} $$

Defining tip propagation operator, $\sigma_t$

$$ \sigma_t = \left[ \begin{array}{ccc} {}_60 & \cdots & {}^i\phi_{n,n-1} \end{array} \right] $$

yields Jacobian by premultiplying both sides of (1)

$$ \mathcal{J} \triangleq \sigma_t\phi H $$

The derivative of (1) provides the link accelerations:

$$ \alpha = \phi(H\ddot{\theta} + a) \tag{2} $$

where $a$ is the the stacked up bias acceleration term. Bias acceleration of link $k$ is defined as

$$ {}^ia_k = \left[ \begin{array}{cc} ({}^i\widehat{\omega}_{k-1}{}^i\omega_k)^T & ({}^i\widehat{\omega}_{k-1}^2{}^i\ell_{k-1,k})^T \end{array} \right]^T $$

Spatial forces are calculated from tip to base. Therefore, $\phi^T$, in this case, becomes the propagation operator.

$$ f = \phi^T(M\alpha + b + \sigma_t^TF_t) \tag{3} $$

where $F_t$ is the tip forces and $b$ is the stacked up spatial bias forces. The definition of spatial bias forces acting on link $k$ is given as

$$ b_k = \left[ \begin{array}{cc} (\widehat{\omega}_k\mathcal{I}_k\omega_k)^T & (m_k\widehat{\omega}_k^2\ell_{k,c})^T \end{array} \right]^T $$

We obtain the applied torques if we project the spatial force vector given in (3) onto the axis of motion.

$$ \mathcal{T} = H^Tf \tag{4} $$

Substituting (3) in (4), we get the equation of motion as

$$ \mathcal{T} = \mathcal{M}\ddot{\theta} + C + \mathcal{J}^TF_t \tag{5} $$

where

$$ \mathcal{M} \triangleq H^T\phi^TM\phi H $$

$$ C \triangleq H^T\phi^TM\phi a + H^T\phi^Tb $$

Hence, forward dynamical model is obtained as

$$ \ddot{\theta} = \mathcal{M}^{-1}(\bar{\mathcal{T}} - \mathcal{J}^TF_t) \tag{6} $$

where

$$ \bar{\mathcal{T}} \triangleq \mathcal{T} - C $$

### C. Calculation of the tip forces

Forward dynamical model given in (6) is obtained for multiple manipulators forming a tree structure. This means that the manipulators do not form loops. In order to obtain the dynamical model of the cooperating manipulators, we need to consider a common payload forming loops or closed kinematic chains. In this case, the tip forces need to be calculated. Let us first take a look at the kinematic constraint due to holding the common load. As displayed at Figure 1, the idea is to propagate the tip velocities to a common point.

$$ AV_t = V_c \tag{7} $$

Kinematic constraint given by equation (7) has a dual pair on the dynamical side:

$$ F_t = A^TF_c \tag{8} $$

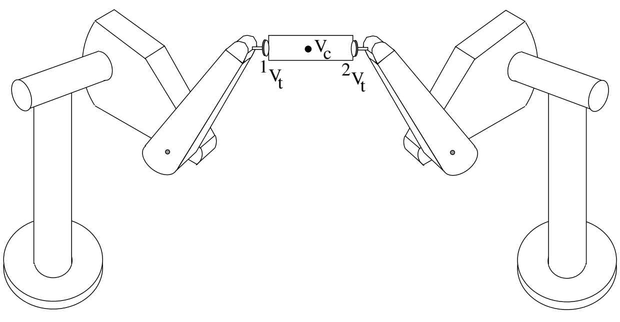


Fig. 1. Common payload

First we go back to the case where there are no loops. Since the closure constraints are not included in this case, the tip forces are taken as zero, i.e., $F_t = 0$. Let us call the joint accelerations of such case “free joint accelerations” and denote them by $\ddot{\theta}^f$. This will introduce an error $\ddot{\theta}^e = \ddot{\theta} - \ddot{\theta}^f$ which will be calculated as well. The following six steps, details of which can be found in [7], lead to the calculation of the constrained tip forces as well as the forward dynamics

1) $\ddot{\theta}^f = \mathcal{M}^{-1}\bar{\mathcal{T}}$
2) $\alpha^f = \mathcal{J}\ddot{\theta}^f$
3) $\Lambda = \mathcal{J}\mathcal{M}^{-1}\mathcal{J}^T$
4) $F_c = (A\Lambda A^T)^{-1}A\alpha^f$
5) $\ddot{\theta}^e = -\mathcal{M}^{-1}\mathcal{J}^T A^T F_c$
6) $\ddot{\theta} = \ddot{\theta}^f + \ddot{\theta}^e$

If we follow these steps, we find

$$\ddot{\theta} = \mathcal{M}^{-1}B\bar{\mathcal{T}} \tag{9}$$

where

$$B = I - \mathcal{J}^T A^T (A\mathcal{J}\mathcal{M}^{-1}\mathcal{J}^T A^T)^{-1} A\mathcal{J}\mathcal{M}^{-1}$$

or, if we call $D = \mathcal{M}^{-1}B$

$$\ddot{\theta} = D\bar{\mathcal{T}} \tag{10}$$

In order for this to work, $\mathcal{J}\mathcal{J}^T$ needs to be full rank. On the other hand, if one of the arms go into a singular configuration then obviously jacobian looses rank. Therefore, the algorithm works under the assumption that no arm will be at singular configuration. Similarly, kinematically deficient manipulators become out of the class of systems that this method can be directly applied.

## III. Dynamics of Cooperating Kinematically Deficient Manipulators

### A. Reduced Order Model

In order to deal with the rank deficiency problem of the jacobian in the case of singular configuration or with the manipulators having less than six DOF, one may suggest to reduce the size of the task space from six. To do that, first we need to find the directions where the tip of the manipulator cannot go. Null space of the transpose of the Jacobian gives us these directions. Based on that we can get the coordinate transformation to reduced task space.

$$R = \mathcal{N}(\mathcal{N}(\mathcal{J}^T)^T)^T \tag{11}$$

Using equation (11) we can replace the Jacobian by $J_r$ as

$$J_r = RJ \tag{12}$$

This is an orthogonal transformation yielding the inverse transformation as

$$J = R^T J_r \tag{13}$$

The six step forward dynamic calculations are updated by replacing $A_r$ with A, where

$$A_r = AR \tag{14}$$

The drawback of this method is with the calculation of the null space. Not only that calculating null space is expensive, considering that it needs to be done at every loop in the iterative method like the one proposed here, but also Singular Value Decomposition (SVD) based numerical methods introduce instability due to the fact that singular vectors are not unique, and may introduce discontinuity.

### B. Pseudo Joints

An easy to implement and computationally efficient alternative approach, is to calculate link internal torques. First we assume as if there were extra joints and then we have to calculate the torques to keep those joints at zero angle at all times as displayed in figure2.

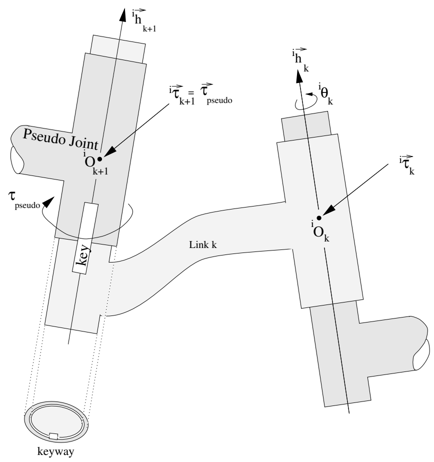


Fig. 2. Pseudo joint in the form of a joint constrained by a key-bushing mechanism

First kinematic analysis needs to be done to decide at what location of which link pseudo joint to be placed in what direction. This analysis is usually straight forward and easy enough to decide by visual inspection of the manipulator. In the more complicated cases, forward kinematic model is obtained and augmented jacobian is desired to be full rank.

We first need to obtain a linear operator dividing the joint space into two sub spaces; real joints and pseudo joints. Let S do that. S can be obtained easily by reordering the rows of $n \times n$ identity matrix, where n is the total DOF including pseudo joints. The rearranged form of the inverse dynamics equation 10 becomes:

$$S\ddot{\theta}_{augmented} = SDS^{-1}\, S\mathcal{T}_{augmented} \tag{15}$$

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\theta}_p \end{bmatrix} = \begin{bmatrix} d_1 & d_2 \\ d_3 & d_4 \end{bmatrix} \begin{bmatrix} \bar{\mathcal{T}} \\ \bar{\mathcal{T}}_p \end{bmatrix} \tag{16}$$

since, $\ddot{\theta}_p = 0$

$$\ddot{\theta} = (d_1 - d_2 d_4^{-1} d_3)\bar{\mathcal{T}} \tag{17}$$

## IV. Example

## V. Dynamics of a Two-Wheeled Cart

As shown in Figure 3, the system consists of two independently actuated wheels that are connected by a rod. From robotics point of view, this can be considered as two arms holding a common load.

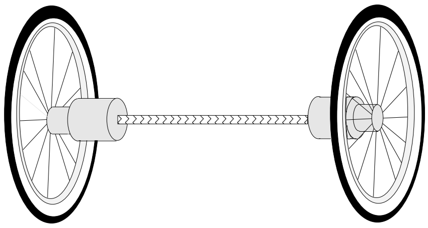

Fig. 3. Two-wheel cart

As in the previous example, each wheel is modeled as a one-link mechanism with 3 DOF joint. Each actuator introduces 1 DOF at the wheel center. As it will be mathematically justified, although there are 8 DOF in total, only two of them are independent. Arm assignment according to this configuration is shown in Figure 4.

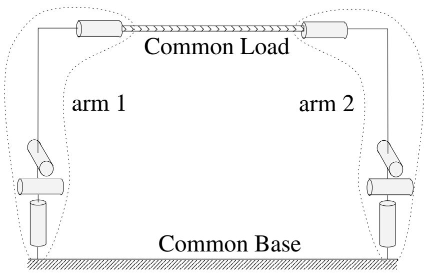


Fig. 4. Two-wheel cart

Figure 5 shows how pseudo joints are located in the example system. Frames are assigned in the same way for both arms as follows: Frame $-1$ is the inertial frame. Base frame, frame $0$, is attached to the ground. Frame $1$ rotates with three-DOF joint, and its origin coincides with that of base frame. Frame $2$ is attached to the first pseudo joint, and frame $3$ is attached to the second pseudo joint. Frame $4$ is on the actuator, and frame $5$, tip frame, represents the end-effector. In this case, frame $4$ and tip frame coincides.

Kinematic constraint of pure rolling is

$$\begin{bmatrix} V_{b-} \\ V_{b+} \end{bmatrix} = \begin{bmatrix} I \\ I \end{bmatrix} V_b \tag{18}$$

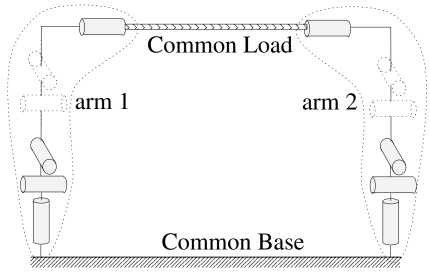


Fig. 5. Two-wheel cart

where

$$V_{b-} = \begin{bmatrix} {}^1V_{b-} \\ {}^2V_{b-} \end{bmatrix} \qquad V_{b+} = \begin{bmatrix} {}^1V_{b+} \\ {}^2V_{b+} \end{bmatrix}$$

Another kinematic constraint which needs to be mentioned comes from rigid grasp. Because two arms are rigidly linked through common load, one can pick any point along the common load and set the velocities propagated from both tip points equal. Let $c$ be a point at the center of the common load, and the spatial velocity at $c$ be $V_c$.

$$V_t = AV_c \tag{19}$$

where

$$A = \begin{bmatrix} {}^1\phi_{t,c} \\ {}^2\phi_{t,c} \end{bmatrix}$$

Although, in this example, vectors from frame $1$ to base frame and from frame $4$ to tip frame are zero vectors, in general it is not the case. (Puma arm, shown in Figure **??**, can be given as an example to this.) In order to keep the derivation general, let us consider the following base and tip transition operators respectively.

$${}^i\sigma_b = \begin{bmatrix} {}^i\phi_{1,0} \\ 0_{\{6\times 6\}} \\ \vdots \\ 0_{\{6\times 6\}} \end{bmatrix}$$

$${}^i\sigma_t = \begin{bmatrix} 0_{\{6\times 6\}} & \cdots & 0_{\{6\times 6\}} & {}^i\phi_{n,n-1} \end{bmatrix}$$

If base velocities and joint relative velocities are known, spatial velocities of the joints with respect to an inertial frame can be obtained by using velocity propagation.

$$V = \phi H\dot{\theta} + \phi\sigma_b V_{b_-} \tag{20}$$

Accelerations are obtained from a similar propagation equation.

$$\alpha = \phi(H\ddot{\theta} + a + \sigma_b\alpha_{b-}) \tag{21}$$

Here, $a$, the bias acceleration term, is augmented as in (22) so that uniform gravity is included in the dynamics.

$$a \leftarrow a + \sigma_b g \tag{22}$$

Since Jacobian, $\mathcal{J}$, is defined as

$$\mathcal{J} = \sigma_t\phi H \tag{23}$$

tip velocities of arms can be calculated as:

$$\begin{aligned} V_t &= \sigma_t V \\ &= \mathcal{J}\dot{\theta} + \phi_{t,b} V_{b_-} \end{aligned} \quad (24)$$

where

$$\phi_{t,b} = \sigma_t \phi \sigma_b$$

Dynamical constraints are the dual pairs of kinematic constraints given by (18) and (19).

$$\begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} F_{b-} \\ F_{b+} \end{bmatrix} = F_b \quad (25)$$

$$A^T F_t = F_c \quad (26)$$

where

$$F_c = M_c \alpha_c + b_c$$

(26) can be solved for $F_t$.

$$F_t = A^{\dagger^T} F_c + \widetilde{A}^T F_{ta} \quad (27)$$

where $\widetilde{A}$ is the annihilator of $A$, one possible choice is

$$\widetilde{A} = \begin{bmatrix} {}^1\phi_{c,t} & -{}^2\phi_{c,t} \end{bmatrix}$$

Spatial forces of all the links are found by propagating the tip spatial forces through base.

$$F = \phi^T (M\alpha + b) + \phi^T \sigma_t^T F_t \quad (28)$$

Substituting (27) in (28)

$$F = \phi^T (M\alpha + b) + (A^\dagger \sigma_t \phi)^T F_c + (\widetilde{A}\sigma_t\phi)^T F_{ta} \quad (29)$$

Next, (21) is substituted.

$$\begin{aligned} F = \phi^T M \phi H \ddot{\theta} + \phi^T M \phi a + \phi^T M \phi \sigma_b \alpha_{b-} + \\ (A^\dagger \sigma_t \phi)^T F_c + (\widetilde{A}\sigma_t\phi)^T F_{ta} \end{aligned} \quad (30)$$

Projecting $F$ onto axes of rotation, applied torques are found.

$$\mathcal{T} = H^T F \quad (31)$$

$$\mathcal{T} = \mathcal{M}\ddot{\theta} + \mathcal{C} + \mathcal{L}\alpha_{b-} + \mathcal{B}^T F_{ta} \quad (32)$$

where

$$\begin{aligned} \mathcal{M} &= H^T \phi^T M \phi H \\ \mathcal{C} &= H^T \phi^T M \phi a + (A^\dagger \sigma_t \phi H)^T F_c \\ \mathcal{L} &= H^T \phi^T M \phi \sigma_b \\ \mathcal{B} &= \widetilde{A} \sigma_t \phi H \end{aligned}$$

Equation of motion can be obtained from (32).

$$\ddot{\theta} = \mathcal{M}^{-1}(\mathcal{T} - \mathcal{C} - \mathcal{L}\alpha_{b-} - \mathcal{B}^T F_{ta}) \quad (33)$$

In order to solve the forward dynamics problem, $F_{ta}$ needs to be determined. First, using the fact that $\widetilde{A}A = 0$, let us write the following equation combined from (19) and (24).

$$\widetilde{A}(\mathcal{J}\dot{\theta} + \phi_{t,b} V_{b-}) = 0 \quad (34)$$

(34) can be differentiated to obtain

$$\widetilde{A}(\mathcal{J}\ddot{\theta} + \phi_{t,b}\alpha_{b-} + \dot{\phi}_{t,b} V_{b-} + \sigma_t \phi a + a_t) + \dot{\widetilde{A}}(\mathcal{J}\dot{\theta} + \phi_{t,b} V_{b-}) = 0 \quad (35)$$

$F_{ta}$ can be solved after substituting (33) in (35).

$$F_{ta} = G_a \mathcal{T} + G_b \dot{\theta} + G_c \alpha_{b-} + G_d V_{b-} + G_e \quad (36)$$

where

$$\begin{aligned} G_a &= -(\widetilde{A}\mathcal{J}\mathcal{M}^{-1}\mathcal{B}^T)^{-1}\widetilde{A}\mathcal{J}\mathcal{M}^{-1} \\ G_b &= -(\widetilde{A}\mathcal{J}\mathcal{M}^{-1}\mathcal{B}^T)^{-1}\dot{\widetilde{A}}\mathcal{J} \\ G_c &= (\widetilde{A}\mathcal{J}\mathcal{M}^{-1}\mathcal{B}^T)^{-1}\widetilde{A}(\mathcal{J}\mathcal{M}^{-1}\mathcal{L} - \phi_{t,b}) \\ G_d &= -(\widetilde{A}\mathcal{J}\mathcal{M}^{-1}\mathcal{B}^T)^{-1}(\widetilde{A}\dot{\phi}_{t,b} + \dot{\widetilde{A}}\phi_{t,b}) \\ G_e &= (\widetilde{A}\mathcal{J}\mathcal{M}^{-1}\mathcal{B}^T)^{-1}(\mathcal{C} - \widetilde{A}(\sigma_t \phi a + a_t)) \end{aligned}$$

Hence, forward dynamics problem is solved.

## VI. Conclusion

Here we demonstrated the methodology associated with dynamical modeling of cooperating kinematically deficient manipulators. This methodology utilizes what we introduced as *Pseudo Joint* which may look as if it complicates the calculations by increasing the DOF of the system but in fact, when compared with alternative approaches, this increase in the DOF has much less computational burden overall. We demonstrated the use of this algorithm on two-wheeled cart problem.

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