An Adaptive Grey Fuzzy PID Controller With Variable Prediction Horizon

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***Abstract*—The concept of grey system theory, which has a certain prediction capability, offers an alternative approach for various kinds of conventional control methods, such as PID control and fuzzy control. For instance, grey PID type fuzzy controller designed in this paper, can predict the future output values of the system accurately. However, the forecasting step-size of the grey controller determines the forecasting value. When the step-size of the grey controller is large, it will cause over compensation, resulting in a slow system response. Conversely, a smaller step-size will make the system respond faster but cause larger overshoots. In order to obtain a better controller performance, another fuzzy controller is designed for changing the step-size of the grey controller. The value of the forecasting step-size is optimized according to the values of error and the derivative of the error. Moreover, the output of the grey controller is uptaded using the prediction error for better controller performance. It is clear that the proposed adaptive PID type fuzzy controller is effective in controlling such a non-linear system by changing the prediction horizon adaptively for real-time working.**

I. INTRODUCTION

Grey system theory was first introduced in early eighties by Professor Deng Ju-long from China [1]. The theory has since then become quite popular with its ability to deal with the systems that have partially unknown parameters. It is therefore a good candidate to real-time control systems.

During the last two decades, grey system theory has developed rapidly and caught the attention of researchers with successful real-time practical applications. It has been applied to analysis, modeling, prediction, decision making and control of various systems such as social, economic, financial, scientific and technological, agricultural, industrial, transportation, mechanical, meteorological, ecological, geological, medical, military, etc., systems [2].

In control theory, a system can be defined with a color that represents the amount of clear information about that system. For instance, a system can be called as a black box if its internal characteristics or mathematical equations that describe its dynamics are completely unknown. On the other hand if the description of the system is, completely known, it can be named as a white system. Similarly, a system that has both known and unknown information is defined as a grey system.

In real life, every system can be considered as a grey system because there are always some uncertainties. Due to noise from both inside and outside of the system of our concern (and the limitations of our cognitive abilities!), the information we can reach about that system is always uncertain and limited in scope [3].

There are many situations in industrial control systems that the control engineer faces the difficulty of incomplete or insufficient information. The reason for this is due to the lack of modeling information or the fact that the right observation and control variables have not been employed. For instance, the data collected from a motor control system always contains some grey characteristics due to the time-varying parameters of the system and the measurement difficulties. Similarly, it is difficult to forecast the electricity consumption of a region accurately because of various kinds of social and economic factors. These factors are generally random and make it difficult to obtain an accurate model.

The traditional grey predictor structure uses a fixed prediction horizon [4]. A grey predictor with a small fixed forecasting step-size will make the system respond faster but cause larger overshoots. Conversely, the bigger step-size of the grey predictor will cause over compensation, resulting in a slow system response. In order to obtain a fast system respond with a little overshoot, the step-size of the grey predictor can be changed adaptively. In the literature of the grey system theory, there are some methods that tune the step-size of the grey predictor according to the input state of the system [5]. In order to determine the appropriate forecasting step-size, some online rule tuning algorithms using a fuzzy inference system have been proposed for the control of an inverted pendulum, fuzzy tracking method for a mobile robot and non-minimum phase systems. [4], [6], [7]. In another paper, a Sugeno type fuzzy inference system has been proposed for large time delay systems [8]. In this work, a similar but simpler approach is proposed.

II. FUNDAMENTAL CONCEPTS OF GREY SYSTEM THEORY

A. Grey System Modeling

Grey numbers, grey algebraic and differential equations, grey matrices and their operations are used to deal with grey systems. A grey number is such a number whose value is not known exactly but it takes values in a certain range. Grey numbers might have only upper limits, only lower limits or both. Grey algebraic and differential equations, grey matrices all have grey coefficients.

B. Generations of Grey Sequences

The main task of grey system theory is to extract realistic governing laws of the system using available data. This process is known as as the generation of the grey sequence [2].

It is argued that even though the available data of the system, which are generally white numbers, is too complex or chaotic, they always contain some governing laws. If the randomness of the data obtained from a grey system is somehow smoothed, it is easier to derive the any special characteristics of that system.

For instance, the following sequence that represents the speed values of a motor might be given:

$$X^{(0)} = (820, 840, 835, 850, 890)$$

It is obvious that the sequence does not have a clear regularity. If accumulating generation is applied to original sequence, $X^{(1)}$ is obtained which has a clear growing tendency.

$$X^{(1)} = (820, 1660, 2495, 3345, 4235)$$

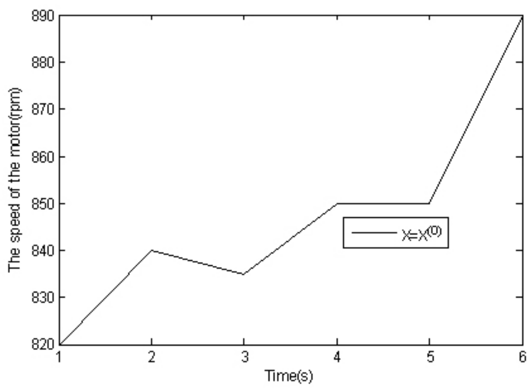

Fig. 1. The original data set

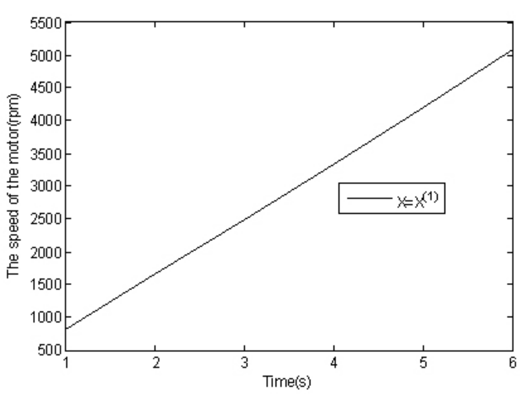

Fig. 2. The accumulated data set

C. GM(n,m) Model

In grey systems theory, GM(n,m) denotes a grey model, where n is the order of the difference equation and m is the number of variables. Although various types of grey models can be mentioned, most of the previous researchers have focused their attention on GM(1,1) model in their predictions because of its computational efficiency. It should be noted that in real time applications, the computational burden is the most important parameter after the performance.

D. GM(1,1) Model

GM(1,1) type of grey model is most widely used in the literature, pronounced as “Grey Model First Order One Variable”. This model is a time series forecasting model. The differential equations of the GM(1,1) model have time-varying coefficients. In other words, the model is renewed as the new data become available to the prediction model.

The GM(1,1) model can only be used in positive data sequences [9]. In this paper, a non-linear liquid level tank is considered. It is obvious that the liquid level in a tank is always positive, so that GM(1,1) model can be used to forecast the liquid level.

In order to smooth the randomness, the primitive data obtained from the system to form the GM(1,1) is subjected to an operator, named Accumulating Generation Operation (AGO)[9], described above. The differential equation (i.e. GM(1,1)) thus evolved is solved to obtain the n-step ahead predicted value of the system. Finally, using the predicted value, the inverse accumulating operation (IAGO) is applied to find the predicted values of original data.

Consider a single input and single output system. Assume that the time sequence $X^{(0)}$ represents the outputs of the system:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\right), n \geq 4 \quad (1)$$

where $X^{(0)}$ is a non-negative sequence and n is the sample size of the data. When this sequence is subjected to the Accumulating Generation Operation (AGO), the following sequence $X^{(1)}$ is obtained. It is obvious that $X^{(1)}$ is monotone increasing.

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)\right), n \geq 4 \quad (2)$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, 3....., n \quad (3)$$

The generated mean sequence $Z^{(1)}$ of $X^{(1)}$ is defined as:

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)\right) \quad (4)$$

where $z^{(1)}(k)$ is the mean value of adjacent data, i.e.

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2, 3, ..., n \quad (5)$$

The least square estimate sequence of the grey difference equation of GM(1,1) is defined as follows [9]:

$$x^{(0)}(k) + az^{(1)}(k) = b \quad (6)$$

The whitening equation is therefore as follows:

$$\frac{dx^1(t)}{dt} + ax^{(1)}(t) = b \tag{7}$$

In above, $[a, b]^T$ is a sequence of parameters that can be found as follows:

$$[a, b]^T = (B^T B)^{-1} B^T Y \tag{8}$$

where

$$Y = \left[x^{(0)}(2), x^{(0)}(3), ..., x^{(0)}(n)\right]^T \tag{9}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{10}$$

According to equation (8.22), the solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a} \tag{11}$$

To obtain the predicted value of the primitive data at time (k+1), the IAGO is used to establish the following grey model.

$$x_p^{(0)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak}(1 - e^a) \tag{12}$$

and the predicted value of the primitive data at time (k+H):

$$x_p^{(0)}(k+H) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-a(k+H-1)}(1 - e^a) \tag{13}$$

The parameter $(-a)$ in the GM(1,1) model is called "development coefficient" which reflects the development states of $X_p^{(1)}$ and $X_p^{(0)}$. The parameter b is called "grey action quantitiy" which reflects changes contained in the data because of being derived from the background values [2].

E. GM(1,1) Rolling Model

GM(1,1) rolling model is based on the forward data of sequence to build the GM(1,1). For instance, using $x^{(0)}(k)$, $x^{(0)}(k+1)$, $x^{(0)}(k+2)$ and $x^{(0)}$(k+3), the model predicts the value of the next point $x^{(0)}(k+4)$. In the next steps, the first point is always shifted to the second. It means that $x^{(0)}(k+1)$, $x^{(0)}(k+2)$, $x^{(0)}$(k+3) and $x^{(0)}$(k+4) are used to predict the value of $x^{(0)}$(k+5). This procedure is repeated till the end of the sequence and this method is called rolling check [10].

GM(1,1) rolling model is used to predict the long continuous data sequences such as the outputs of a system, price of a specific product, trend analysis for finance statements or social parameters, etc... In this paper, GM(1,1) rolling model is used to predict the future outputs of the non-linear liquid level system.

III. Combining Fuzzy and PID Type Control

A. Analysis of a Fuzzy Controller

Consider a product-sum type fuzzy controller with two inputs and one crisp output (MISO). Let the inputs to the fuzzy controller be the error e and the rate of change of the error $\dot{e}$, and the output of the fuzzy controller (that is the input to the controlled process) be u. If an analysis of this controller is made, it can be seen that it behaves approximately like a PD controller. We can therefore consider it as a time-varying parameter PD controller [11]. Such a controller is named as a PD type fuzzy controller (PDFC) in the literature. It is well known that if the controlled system is type "0", a P or PD type controller cannot eliminate the steady-state error. Although the use of an integral term in the controller (such as PI controller) can take care of the steady-state error, it can deteriorate the transient characteristics by slowing the response. However, with a PID-type fuzzy controller fast rise times and small overshoots as well as short settling times can be achieved with no steady-state error.

B. PID Type Fuzzy Control

In order to design a PID type fuzzy controller (PIDFC), one can design a fuzzy controller with with three inputs, error, the change rate of error and the integration of the error. Handling the three variables is however, in practice, quite difficult. Besides, adding another input to the controller will increase the number of rules exponentially. This requires more computational effort, leading to larger execution time.

Because of the drawbacks mentioned above, a PID type fuzzy controller consisting of only the error and the rate of change of error is used in the proposed method. This allows PD and PI type fuzzy controllers to work in parallel [11], [12].

An equivalent structure is shown in Fig. 3, where β and α are the weights of PI and PD type controllers, respectively. Similarly, K and K_d are the scaling factors for e and $\dot{e}$, respectively.

As the α/β ratio becomes larger, the effect of the derivative control increases with respect to the integral control [13].

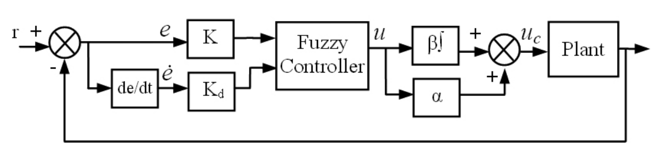

Fig. 3. Block diagram of the PID type fuzzy control system

The output of the controller can be expressed as,

$$u_c = \alpha u + \beta \int u dt \tag{14}$$

This controller is called as PID type fuzzy controller (PIDFC).

IV. Grey PID Type Fuzzy Control

A. Rule Base and Membership Functions

In a conventional fuzzy inference system, an expert, who is familiar with the system to be modeled, decides on the number of rules. The fuzzy PID type control rule base employed in this thesis is shown in Table 1. The membership functions of error, change rate of error and control signal, shown in Fig. 4, are chosen as triangular membership functions.

TABLE I

A GENERAL FUZZY PID TYPE RULE BASE

e-$\dot{e}$	**NL**	**NM**	**NS**	**ZR**	**PS**	**PM**	**PL**
PL	ZR	PS	PM	PL	PL	PL	PL
PM	NS	ZR	PS	PM	PL	PL	PL
PS	NM	NS	ZR	PS	PM	PL	PL
ZR	NL	NM	NS	ZR	PS	PM	PL
NS	NL	NL	NM	NS	ZR	PS	PM
NM	NL	NL	NL	NM	NS	ZR	PS
NL	NL	NL	NL	NL	NM	NS	ZR

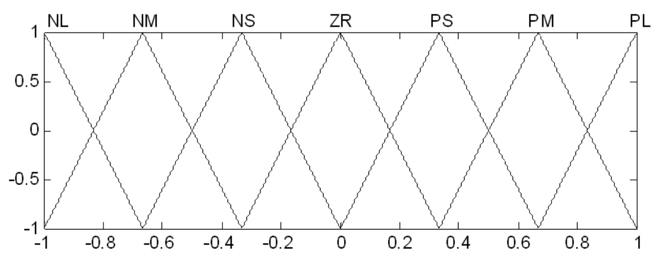

Fig. 4. The membership functions of e, $\dot{e}$ and u.

B. Design of Adaptive Grey PID Type Fuzzy Controller

In most control applications, the control signal is a function of the error present in the system at a prior time. This methodology is called as "delay control". In grey systems theory, prediction error is used instead of current measured error [14]. In similar lines, during the development of the grey PID type fuzzy controller, the prediction error is considered as the error of the system. The block diagram of the grey fuzzy PID control system with a fixed prediction horizon and the adaptive grey PID type fuzzy controller with a variable prediction horizon proposed in this paper are showed in Fig. 5 and Fig. 6, respectively.

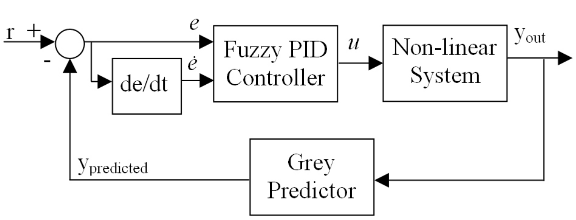

Fig. 5. Block diagram of the grey fuzzy PID control system with a fixed prediction horizon

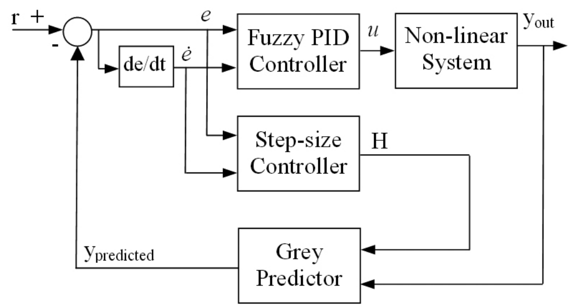

Fig. 6. Block diagram of the adaptive grey fuzzy PID control system with a variable prediction horizon

In order to adapt the forecasting step-size H to different states of the controller changing with error and the derivative of the error, another fuzzy controller is designed. The inputs of this fuzzy controller are e and $\dot{e}$. The output variable is forecasting step-size H. Triangle membership functions are used for fuzzification process. The fuzzy variables e and $\dot{e}$ are partitioned into 5 subsets(NL, NS, ZR, PS, PL) and the output variable H is partitioned into 5 subsets(VS, S, MD, B, VB). The range of e, $\dot{e}$ and H is considered as [-0.4;1], [-0.05;0.05] and [0;60], respectively.

TABLE II

A GENERAL RULE BASE FOR FUZZY STEP-SIZE CONTROLLER

H Step-size		e				
		PL	**PS**	**ZR**	**NS**	**NL**
$\dot{e}$	**PL**	VB	VB	B	MD	VS
	PS	VB	B	MD	S	S
	ZR	S	S	S	S	MD
	NS	S	S	MD	MD	VB
	NL	VS	MD	B	B	VB

V. Description of Controlled Object

A model of a nonlinear liquid-level system will be obtained in this part of the paper [15]. Fig. 7 shows a simple system, the objective of which is to control the level of the liquid in a tank by adjusting the input flow rate in an effective way.

In this system, Q_{in} and Q_{out} are the maximum liquid flow rates in m^3/s for input and outlet, respectively.

The controlled input liquid flow rate q_{in} is given by:

$$q_{in} = Q_{in} \sin(\phi(t)) \qquad \phi(t) \in [0, \pi/2] \tag{15}$$

The output liquid flow rate q_{out} (that equals Q_{out} since no control is applied) is defined as:

$$q_{out} = a_{out}\sqrt{2gh(t)} \tag{16}$$

where a_{out} is surface area of the outlet and g is the gravitational constant.

The output variable h, which is the level of the liquid, is calculated as:

$$h(t) = h(0) + \frac{1}{A}\int_0^t (q_{in}(\tau) - q_{out}(\tau))d\tau \tag{17}$$

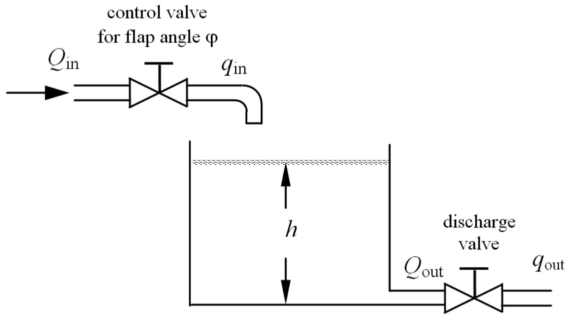

Fig. 7. A nonlinear liquid-level system

where, A is the surface area of the tank.
The numerical values used in this paper are:
$A = 1m^2$,
$a_{out} = 0.01m^2$,
$Q_{in} = 0.12m^3/s$, and
$h(0) = 0$.

VI. Non-Linear Model Simulations

In this section, computer simulated dynamic responses are performed on the non-linear liquid level system that was modeled in the previous section. The non-linear system differential equations are simultaneously solved by using the Dormand-Prince algorithm. The numerical values used in this paper are $K = 1$ and $K_d = 0.1$. The simulation sample time T is equal to 0.4s.

Fig. 8 shows the response of the model to PDFC, PIFC and PIDFC. As can be seen the system response is very fast but there is a steady-state error with PDFC. With PIFC, the system does not have a steady-state error but a big overshoot and a slow response. The steady-state error of the system can be eliminated with a fast response using PIDFC.

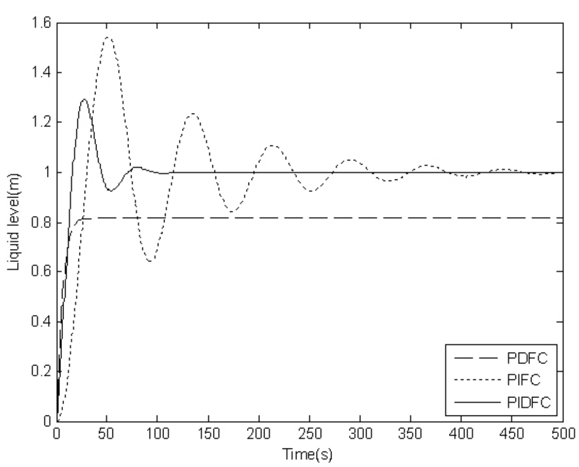

Fig. 8. Step responses of the system to PDFC ($\alpha = 8.2$, $\beta = 0$), PIFC ($\alpha = 0$, $\beta = 0.25$) and PIDFC ($\alpha = 3.2$, $\beta = 0.6$)

Fig. 9 shows the step responses of the system to PIDFC, grey PIDFC with a fixed H and grey PIDFC with variable step-size. With grey PIDFC using a variable step-size, the system has a fast rise time and a reasonable overshoot. However, a switching characteristic can be seen on the response of the grey PIDFC with variable step-size.

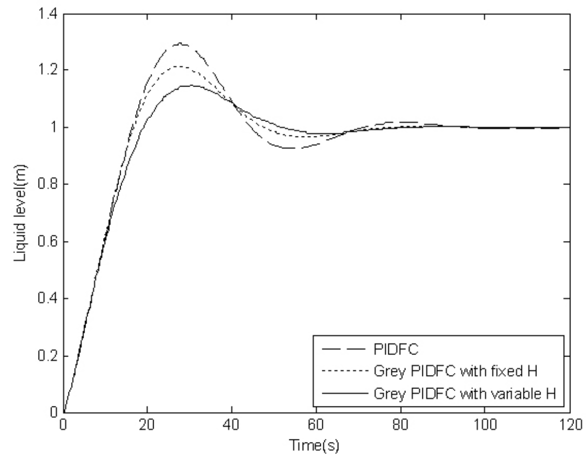

Fig. 9. Step responses of the system to grey PIDFC with a fixed H=20 and grey PIDFC with variable H ($\alpha = 3.2$, $\beta = 0.6$)

Fig. 10 shows the unit step responses of the system to grey PIDFC with a fixed step-size and grey PIDFC with variable step-size with the band-limited white noise at the output measurement. The noise power, which is the height of the power spectral density of the white noise, is equal to 0.0002. The correlation time of the noise is equal to 0.4 sec. Although the response of the conventional grey controller is acceptable, the grey predictor with variable step-size is better under noisy conditions. This indicates that adaptive grey predictive controllers are more robust in real-time systems that are subject to noise from both inside and outside of the system.

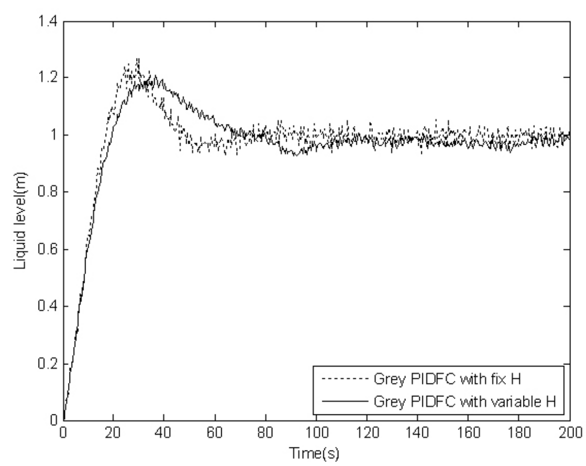

Fig. 10. Step responses of the system to grey PIDFC with a fixed H=20 and grey PIDFC with variable H when there is white noise at the output measurement($\alpha = 3.2$, $\beta = 0.6$)

Fig. 11-12 show the unit step responses of the non-linear liquid level system to grey PIDFC with a fixed step-size and grey PIDFC with variable step-size when the surface area of the outlet a_{out} is reduced to its 0.2 times its normal value in 100^{th} second and reduced to zero between 100^{th} and 110^{th} seconds, respectively.

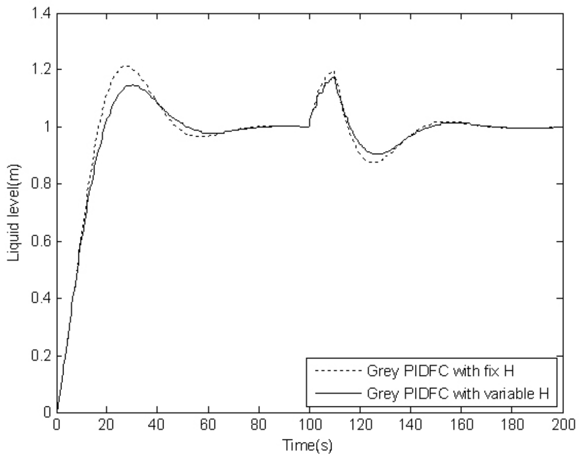

Fig. 11. Step responses of the system to grey PIDFC with a fixed H=20 and grey PIDFC with variable H when the surface area of the outlet a_{out} is reduced to its 0.2 times its normal value ($\alpha = 3.2$, $\beta = 0.6$)

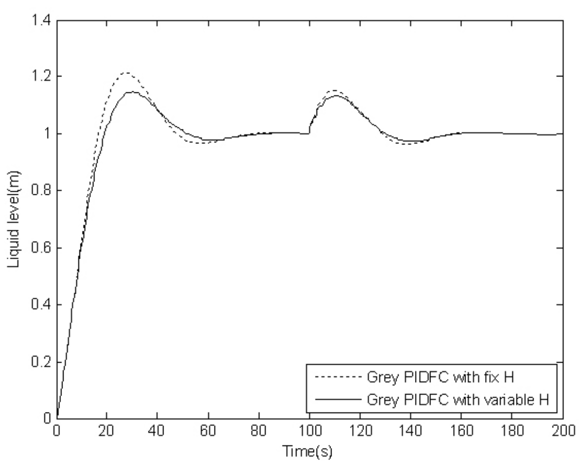

Fig. 12. Step responses of the system to grey PIDFC with a fixed H=20 and grey PIDFC with variable H when the surface area of the outlet a_{out} is reduced to zero for 10 seconds ($\alpha = 3.2$, $\beta = 0.6$)

VII. Conclusion

This paper proposes a grey PIDFC with a variable prediction horizon for a nonlinear liquid level system. The simulation results show that the proposed method not only reduces the overshoot and the rise time but also maintain a better disturbance rejection. In real life, there are always some uncertainties because an accurate mathematical model of a physical system cannot generally be defined. Noise that exists in various stages of the system is an additional problem. The proposed adaptive grey PIDFC has the ability to handle these difficulties.

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