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Neuro-Fuzzy Control of Antilock Braking System Using Variable-Structure-Systems-based Learning Algorithm

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Abstract— **A neuro-fuzzy adaptive control approach for nonlinear systems with model uncertainties is proposed. The implemented control scheme consists of a proportional plus derivative controller that is provided both to guarantee global asymptotic stability in compact space and as an inverse reference model of the response of the controlled system. Its output is used as an error signal by an on-line learning algorithm to update the parameters of a neuro-fuzzy feedback controller. The latter is able to gradually replace the conventional controller from the control of the system. The proposed learning algorithm makes direct use of the variable structure systems theory and establishes a sliding motion in terms of the neuro-fuzzy controller parameters. An integrating term has been additionally applied to the overall control signal of the two controllers and the performance of the control scheme has been tested on the wheel slip control problem within an antilock breaking system model. The analytical claims have been justified under the existence of model uncertainties and large initial errors.**

Keywords-antilock braking systems; variable structure systems; neuro-fuzzy networks; adaptive systems

I. INTRODUCTION

The design of a wheel slip regulating controller is not an easy task and many difficulties arise because of the highly nonlinear and uncertain structure of antilock braking system (ABS). One of the shortcomings is that for optimal performance the controller must operate at an unstable equilibrium point. A small perturbation of the controller input may induce a drastic change in the output. The performance of ABS is not always satisfactory due to the existing high dependence of the system parameters on the road conditions that vary over a wide range. At present, affordable sensors that are capable to identify accurately the road surface, and provide these data to the ABS controller do not exist and the available sensor signals are usually highly uncertain and noisy [1]. In addition, an accurate measurement of vehicle's absolute velocity is required to calculate the wheel slip and to solve this problem a number of estimators have been proposed throughout the literature. For instance, in [2], recursive least squares method is used to estimate the real time vehicle velocity.

All these factors hamper the development of an accurate mathematical model of ABS and therefore advanced control design techniques are required for the wheel slip controller that are able to cope with ill-defined systems, containing uncertainties in their dynamic models. Sliding mode control is a preferable option, as it guarantees the robustness of the system against changing operation conditions. Kachroo and Tomizuka proposed a sliding mode controller that can maintain the wheel slip at any desired value [3]. Unsal and Kachroo proposed a sliding mode observer to track the reference wheel slip, and a proportional plus integral (PI)-like controller is used to reduce the chattering problem [4]. Besides the sliding mode control approach, several control schemes have been proposed including fuzzy logic, neural networks and hybrid control approaches. For instance, a self learning fuzzy controller, combined with a sliding mode controller have been designed in [5] where the proposed tuning algorithms for the controller have been derived in accordance with the Lyapunov's stability theory.

In the present paper a new neuro-fuzzy adaptive control approach is developed for nonlinear dynamical systems, coupled with unknown dynamics, modeling errors, and various sorts of disturbances. The implemented control scheme consists of a conventional proportional plus derivative (PD) controller and a neuro-fuzzy feedback controller. An on-line learning algorithm that makes direct use of the variable structure systems theory and establishes a sliding motion in terms of the neuro-fuzzy controller parameters has been applied, leading the learning error toward zero. The convergence of the algorithm is established and the conditions are given. An integrating term has been additionally applied to the overall control signal of the two controllers and the proposed approach has been tested using simulations on the wheel slip control problem. The analytical claims have been justified under the presence of uncertainty and large initial errors.

The paper is organized as follows. Section II starts with an introduction to the quarter vehicle model describing longitudinal motion of the vehicle and angular motion of the wheel under braking, then continues with the proposed neuro-fuzzy adaptive control approach. Then, the developed new on-line variable structure systems-based method for parametric adaptation of fuzzy rule-based neural networks with a scalar output is presented. Section III is devoted to the obtained results from simulations, and the concluding remarks are given in section IV.

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II. THE NEURO-FUZZY ADAPTIVE CONTROL APPROACH

A. *The System to be Controlled*

The free body diagram of the quarter vehicle model describing longitudinal motion of the vehicle and angular motion of the wheel under braking is presented in Fig. 1, [6].

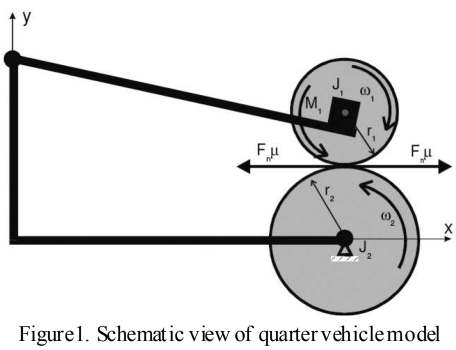

Figure1. Schematic view of quarter vehicle model

Although the model is quite simple, it preserves the fundamental characteristics of an actual system. In deriving the dynamic equations of the system, several assumptions have been made. First, only longitudinal dynamics of the vehicle has been considered. The lateral and vertical motions have been neglected. Furthermore, it has been assumed that there is no interaction between the four wheels of the vehicle [7].

Regarding the model, there are three torques acting on the upper wheel: braking torque, friction torque in the upper bearing and the friction torque among the wheels. Two torques are acting on the lower wheel: the friction torque in the lower bearing and the friction torque between the wheels. The parameters of the system are presented in Table 1.

TABLE I. NOMENCLATURE

Name	Description
ω_1	Angular velocity of the upper wheel
ω_2	Angular velocity of the lower wheel
T_B	Braking torque
r_1	Radius of the upper wheel
r_2	Radius of the lower wheel
J_1	Moment of inertia of the upper wheel
J_2	Moment of inertia of the lower wheel
d_1	Viscous friction coefficient of the upper wheel
d_2	Viscous friction coefficient of the lower wheel
F_n	Total normal load
μ	Road adhesion coefficient
λ	Wheel slip
λ_d	Desired slip
F_t	Road friction force
M_{10}	Static friction of the upper wheel
M_{20}	Static friction of the lower wheel
M_g	Moment of gravity acting on balance lever

- During deceleration, a braking torque is applied to the upper wheel, which causes wheel speed to decrease. According to Newton's second law, the equations of the motion of the system can be written as follows:

$$J_1\dot{\omega}_1 = F_t r_1 - \left(d_1\omega_1 + M_{10} + T_B\right) \quad (1)$$

$$J_2\dot{\omega}_2 = -\left(F_t r_2 + d_2\omega_2 + M_{20}\right) \quad (2)$$

F_t in (1) and (2) stands for the road friction force which is given by Coulomb law:

$$F_t = \mu(\lambda) F_n \quad (3)$$

F_n is calculated by the following equation:

$$F_n = \frac{d_1\omega_1 + M_{10} + T_B + M_g}{L\left(\sin\phi - \mu(\lambda)\cos\phi\right)} \quad (4)$$

where L is the distance between the contact point of the wheels and the rotational axis of the balance lever of the upper wheel and ϕ is the angle between the normal in the contact point and the line L.

Under normal operating conditions, the rotational velocity of the wheel would match the forward velocity of the car. When the brakes are applied, braking forces are generated at the interface between the wheel and road surface, which causes the wheel speed to decrease. As the braking force at the wheel increases, slippage will occur between the tire and the road surface. The wheel speed will tend to be lower than vehicle speed. The parameter used to specify the difference between the above two velocities is called wheel slip (λ), and it is defined as follows:

$$\lambda = \frac{r_2\omega_2 - r_1\omega_1}{r_2\omega_2} \quad (5)$$

While a wheel slip of 0 indicates that the wheel velocity and the vehicle velocity are the same, a ratio of 1 indicates that the tire is not rotating and the wheels are skidding on the road surface, i.e., the vehicle is no longer steerable.

B. *The Control Scheme and the Neuro-fuzzy Network Structure*

The proposed control scheme is depicted on Fig. 2.

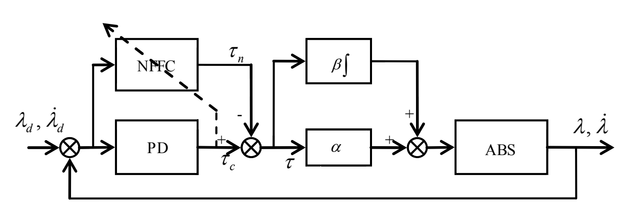

Figure 2. Block diagram of the proposed adaptive neuro-fuzzy control scheme for ABS slip tracking.

A PD controller is provided both as an ordinary feedback controller to guarantee global asymptotic stability in compact space and as an inverse reference model of the response of the system under control. The PD control law is described as follows:

$$\tau_c = k_D\dot{e} + k_P e \quad (6)$$

where $e=\lambda_d-\lambda$ is the feedback error, λ_d is the desired slip value, k_P and k_D are the controller gains.

Consider a neuro-fuzzy network with two inputs and one output used as a feedback controller (the NFFC block on Fig. 2). Its input signals $x_1(t)=e(t)$ and $x_2(t)=\dot{e}(t)$ are associated accordingly with I and J numbers of fuzzy labels which are determined by their corresponding Gaussian membership functions μ.

A fuzzy *if-then* rule base of Takagi-Sugeno type is implemented where the fuzzy sets are included in the premise part only. In this case the corresponding rule R_{ij} can be expressed as:

R_{ij} : *if* x_1 is A_i and x_2 is B_j *then* $f_{ijk}=a_ix_1+b_jx_2+d_{ij}$

where $i=1,...,I$ and $j=1,...,J$.

It is further assumed that the output of each rule consists solely of a constant d_{ij} which is a widely used simplification.

Each membership function is described by two parameters – the center c and the distribution σ which are among the tunable parameters of the above fuzzy-neural structure.

The strength of the rule R_{ij} is obtained as a *T*-norm of the membership functions in the premise part (by using a multiplication operator):

$$W_{ij}=\mu_{A_i}(x_1)\mu_{B_j}(x_2) \quad (7)$$

The output signal of the fuzzy-neural network $\tau_n(t)$ is calculated as a weighted average of the output of each rule:

$$\tau_n(t)=\frac{\sum_{i=1}^{i\max}\sum_{j=1}^{j\max}W_{ij}f_{ij}}{\sum_{i=1}^{i\max}\sum_{j=1}^{j\max}W_{ij}}=\sum_{i=1}^{I}\sum_{j=1}^{J}f_{ij}\bar{W}_{ij} \quad (8)$$

where $\bar{W}_{ij}$ is the normalized value of the output signal of the neuron ij from the second hidden layer of the network:

The following vectors have been specified:

$\bar{W}(t)=\left[\bar{W}_{11}(t)\,\bar{W}_{12}(t)...\bar{W}_{21}(t)...\bar{W}_{ij}(t)...\bar{W}_{IJ}(t)\right]^T$ – vector of the normalized output signals of the neurons from the second hidden layer;

$\sigma_A=\left[\sigma_{A_1}\ \sigma_{A_2}\ ...\sigma_{A_i}\ ...\sigma_{A_I}\right]^T$ – vector of the parameters defining the distribution of the Gaussian membership functions relevant to the first input of the network;

$\sigma_B=\left[\sigma_{B_1}\ \sigma_{B_2}\ ...\sigma_{B_j}\ ...\sigma_{B_J}\right]^T$ – vector of the parameters defining the distribution of the Gaussian membership functions relevant to the second input of the neuro-fuzzy network;

$c_A=\left[c_{A_1}\ c_{A_2}\ ...c_{A_i}\ ...c_{A_I}\right]^T$ – vector of the parameters defining the centers of the Gaussian membership functions relevant to the first network input;

$c_B=\left[c_{B_1}\ c_{B_2}\ ...c_{B_j}\ ...c_{B_J}\right]^T$ – vector of the parameters defining the centers of the Gaussian membership functions relevant to the second network input;

The following assumptions have been used in this investigation:

Both, the input signals $x_1(t)$ and $x_2(t)$, and their time derivatives are considered bounded:

$$x_1(t)\le B_x,\ x_2(t)\le B_x,\ \dot{x}_1(t)\le B_{\dot{x}},\ \dot{x}_2(t)\le B_{\dot{x}}\ \forall t \quad (9)$$

where B_x and $B_{\dot{x}}$ are known positive constants.

The vectors defining the distributions and the centers of the membership functions are assumed bounded as follows:

$$\|\sigma_A\|\le B_\sigma,\ \|\sigma_B\|\le B_\sigma,\ \|c_A\|\le B_c,\ \|c_B\|\le B_c \quad (10)$$

where B_σ and B_c are known positive constants.

It is also assumed that, due to the physical constraints, the time variable weight coefficients of the connections between the neurons in the second hidden layer and the output neuron are bounded, i. e.,

$$|f_{ij}(t)|\le B_f \quad (11)$$

for some positive constant B_f.

From (7) to (10) it follows that $0<\bar{W}_{ij}<1$ and it can be easily seen from (8) that $\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}=1$.

The control signal τ is determined as follows:

$$\tau=\tau_c-\tau_n \quad (12)$$

τ and $\dot{\tau}$ are considered as bounded signals, i.e.

$$|\tau(t)|\le B_\tau,\ |\dot{\tau}(t)|\le B_{\dot{\tau}}\quad \forall t \quad (13)$$

where B_τ and $B_{\dot{\tau}}$ are some known positive constants.

The weakness of the approach based on the usage of PD control law is that in many cases it is not possible to remove out the steady state error. To overcome this problem a proportional plus derivative plus integral (PID) controller can be applied. The latter approach has been adopted and implemented in this investigation by adding one common for both (conventional and neuro-fuzzy) controllers integrating term, placed after the summing junction for their output signals (see Fig.2, where the gains α and β can be adjusted appropriately by using a trial and error procedure).

C. *The Sliding Mode Learning Algorithm*

Using the sliding mode control theory principles [8] the zero value of the learning error coordinate $\tau_c(t)$ can be defined as time-varying sliding surface, i.e.,

$$S_c(\tau_n,\tau)=\tau_c(t)=\tau_n(t)+\tau(t)=0 \quad (14)$$

thus the neuro-fuzzy network is trained to become a nonlinear regulator and to obtain the desired response during the tracking-error convergence movement by compensation for the nonlinearity of the plant.

The sliding surface for the nonlinear system under control $S_p(e,\dot{e})$ is defined as

$$S_p(e,\dot{e})=\dot{e}+\chi e \quad (15)$$

with χ being a constant determining the slope of the sliding surface.

Definition: A sliding motion will have place on a sliding manifold $S_c\left(\tau_n,\tau\right)=\tau_c\left(t\right)=0$ after a time t_h, if the condition $S_c(t)\dot{S}_c(t)=\tau_c\left(t\right)\dot{\tau}_c\left(t\right)<0$ is satisfied for all t in some nontrivial semi-open subinterval of time of the form $\left[t,t_h\right)\subset\left(-\infty,t_h\right)$.

It is desired to devise a dynamical feedback adaptation mechanism for the neuro-fuzzy network parameters such that the sliding mode condition of the above definition is enforced.

Theorem 1: If the adaptation law for parameters of the considered neuro-fuzzy network is chosen respectively as:

$$\dot{c}_{A_i}=-\frac{\sigma_{A_i}}{\sigma_A^T\sigma_A}\alpha sign\left(\tau_c\right),\ \dot{c}_{B_j}=-\frac{\sigma_{B_j}}{\sigma_B^T\sigma_B}\alpha sign\left(\tau_c\right) \tag{16}$$

$$\dot{\sigma}_{A_i}=-\frac{s_{A_i}}{s_A^T s_A}\alpha sign\left(\tau_c\right),\ \dot{\sigma}_{B_j}=-\frac{s_{B_j}}{s_B^T s_B}\alpha\, sign\left(\tau_c\right) \tag{17}$$

$$\dot{f}_{ij}=-\frac{\bar{W}_{ij}}{\bar{W}^T\bar{W}}\alpha sign\left(\tau_c\right) \tag{18}$$

where $s_A=\left[s_{A_1}\, s_{A_2}\ldots s_{A_I}\right]^T$, $s_B=\left[s_{B_1}\, s_{B_2}\ldots s_{B_J}\right]^T$, $s_{A_i}=x_1-c_{A_i}$, $s_{B_j}=x_2-c_{B_j}$ and α is a sufficiently large positive constant satisfying the inequality

$$\alpha>\frac{4B_rB_f+B_{\dot{\tau}}}{1-8B_qB_f} \tag{19}$$

then given an arbitrary initial condition $\tau_c\left(0\right)$, the learning error $\tau_c\left(t\right)$ will converge to zero during a finite time t_h which may be estimated as

$$t_h\le\frac{\left|\tau_c\left(0\right)\right|}{\alpha-\left(8B_q+4B_r\right)B_f-B_{\dot{\tau}}}$$

and a sliding motion will have place on $\tau_c\left(t\right)=0$ for all $t>t_h$.

Proof: Consider the following Lyapunov function candidate:

$$V_c=\frac{1}{2}\tau_c^2\left(t\right) \tag{20}$$

The time derivative of V_c is given by

$$\dot{V}_c=\tau_c\dot{\tau}_c=\tau_c(\dot{\tau}_n+\dot{\tau})=\tau_c\left[\frac{d}{dt}\left(\sum_{i=1}^{I}\sum_{j=1}^{J}f_{ij}\bar{W}_{ij}\right)+\dot{\tau}\right]=$$

$$=\tau_c\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\dot{f}_{ij}\bar{W}_{ij}+f_{ij}\dot{\bar{W}}_{ij}\right)+\dot{\tau}\right] \tag{21}$$

It can be easily shown that

$$\dot{\bar{W}}_{ij}=-\bar{W}_{ij}\dot{K}_{ij}+\bar{W}_{ij}\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\bar{W}_{ij}\dot{K}_{ij}\right) \tag{22}$$

where $\dot{K}_{ijk}=2\left(A\dot{A}+B\dot{B}\right)$, $A=\frac{x_1-c_{A_i}}{\sigma_{A_i}}$ and $B=\frac{x_2-c_{B_j}}{\sigma_{B_j}}$.

Then $\dot{V}_c$ can be further expressed as follows:

$$\dot{V}=\tau_c\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\dot{f}_{ij}\bar{W}_{ij}+f_{ij}\left(-2\bar{W}_{ij}K_{ij}+2\bar{W}_{ij}\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}K_{ij}\right)\right]+\dot{\tau}\right\}=$$

$$=\tau_c\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}\dot{f}_{ij}\bar{W}_{ij}-2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}\left(A\dot{A}+B\dot{B}\right)f_{ij}+\right.$$

$$\left.+2\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\bar{W}_{ij}f_{ij}\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}\left(A\dot{A}+B\dot{B}\right)\right]+\dot{\tau}\right\}=$$

$$=\tau_c\left\{-\alpha\, sign(\tau_c)-2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}\, f_{ij}\left(\frac{A}{\sigma_{A_i}^2}\left(\dot{x}_1\sigma_{A_i}+2\alpha\, sign\left(\tau_c\right)\right)+\right.\right.$$

$$\left.+\frac{B}{\sigma_{B_j}^2}\left(\dot{x}_2\sigma_{B_j}+2\alpha\, sign\left(\tau_c\right)\right)\right)+$$

$$+2\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\bar{W}_{ij}f_{ij}\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}\left(\frac{A}{\sigma_{A_i}^2}\left(\dot{x}_1\sigma_{A_i}+2\alpha\, sign\left(\tau_c\right)\right)+\right.\right.$$

$$\left.\left.\left.+\frac{B}{\sigma_{B_j}^2}\left(\dot{x}_2\sigma_{B_j}+2\alpha\, sign\left(\tau_c\right)\right)\right)\right]+\dot{\tau}\right\}=$$

$$=\tau_c\left[-\alpha\, sign\left(\tau_c\right)-2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}f_{ij}r_{ij}-4\alpha\, sign\left(\tau_c\right)\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}f_{ij}q_{ij}+\right.$$

$$\left.+2\tau_n\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}r_{ij}+4\tau_n\alpha\, sign\left(\tau_c\right)\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}q_{ij}+\dot{\tau}\right]=$$

$$=\left[-\alpha-4\alpha\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}q_{ij}\left(f_{ij}-\tau_n\right)\right]\left|\tau_c\right|-$$

$$-\left[2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}r_{ij}\left(f_{ij}-\tau_n\right)+\dot{\tau}\right]\tau_c\le$$

$$\le-\alpha\left|\tau_c\right|+4\alpha\left|\tau_c\right|\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}B_q\left(B_f+B_f\right)+$$

$$+2\left|\tau_c\right|\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}B_r\left(B_f+B_f\right)+B_{\dot{\tau}}\left|\tau_c\right|=$$

$$\le\left|\tau_c\right|\left[-\alpha\left(1-8B_qB_f\right)+4B_rB_f+B_{\dot{\tau}}\right]<0 \tag{23}$$

where r_{ijk} and q_{ijk} are defined as follows:

$$r_{ij}=\frac{A}{\sigma_{A_i}}\dot{x}_1+\frac{B}{\sigma_{B_j}}\dot{x}_2,\ q_{ij}=\frac{A}{\sigma_{A_i}^2}+\frac{B}{\sigma_{B_j}^2},\ \left|r_{ij}\right|\le B_r,\ \left|q_{ij}\right|\le B_q \tag{24}$$

and the positive constants B_r and B_q are bounded by the following inequalities:

$$B_r\le 2B_{\dot{x}}\frac{B_x+B_c}{B_\sigma^2},\ B_q\le 2\frac{B_x+B_c}{B_\sigma^3} \tag{25}$$

The inequality (23) shows that the controlled trajectories of the learning error $\tau_c\left(t\right)$ converge to zero in a stable manner.

It can be shown that such a convergence takes place in finite time. The differential equation that is satisfied by the controlled error trajectories $\tau_c\left(t\right)$ is as follows:

$$\dot{\tau}_c\left(t\right)=-\alpha sign\left(\tau_c\right)-2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}r_{ij}\left(f_{ij}-\tau_n\right)-$$

$$-4\alpha sign\left(\tau_c\right)\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}q_{ij}\left(f_{ij}-\tau_n\right)+\dot{\tau}=$$

$$= -\alpha sign(\tau_c)\left[1+4\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}q_{ij}\left(f_{ij}-\tau_n\right)\right]-$$

$$-2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}r_{ij}\left(f_{ij}-\tau_n\right)+\dot{\tau} \quad (26)$$

For any $t \le t_h$, the solution $\tau_c(t)$ of this equation, with initial condition $\tau_c(0)$ at $t=0$, satisfies

$$\tau_c(t)-\tau_c(0)=\int_0^t \dot{\tau}_c(\varsigma)d\varsigma=\int_0^t\Big\{-\alpha sign\left(\tau_c(\varsigma)\right)\cdot$$

$$\cdot\left[1+4\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}(\varsigma)q_{ij}(\varsigma)\left(f_{ij}(\varsigma)-\tau_n(\varsigma)\right)\right]-$$

$$-2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}(\varsigma)r_{ij}(\varsigma)\left(f_{ij}(\varsigma)-\tau_n(\varsigma)\right)+\dot{\tau}(\varsigma)\Big\}d\varsigma \quad (27)$$

At time $t=t_h$ the solution takes zero value and, therefore,

$$-\tau_c(0)=\int_0^{t_h}\Big\{-\alpha sign\left(\tau_c(0)\right)\left[1+4\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}(t)q_{ij}(t)\left(f_{ij}(t)-\tau_n(t)\right)\right]-$$

$$-2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}(t)r_{ij}(t)\left(f_{ij}(t)-\tau_n(t)\right)+\dot{\tau}(t)\Big\}dt=$$

$$=-\alpha sign\left(\tau_c(0)\right)\Big\{t_h+4\int_0^{t_h}\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}(t)q_{ij}(t)\right.$$

$$\left.\left(f_{ij}(t)-\tau_n(t)\right)\right]dt\Big\}-\int_0^{t_h}\left[2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}(t)r_{ij}(t)\right.$$

$$\left.\left(f_{ij}(t)-\tau_n(t)\right)-\dot{\tau}(t)\right]dt \quad (28)$$

By multiplying both sides of equation (28) by $-sign\left(\tau_c(0)\right)$ the estimate of t_h can be found

$$\left|\tau_c(0)\right|=\alpha t_h+4\alpha\int_0^{t_h}\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}(t)q_{ij}(t)\left(f_{ij}(t)-\tau_n(t)\right)\right]dt+$$

$$+sign\left(\tau_c(0)\right)\int_0^{t_h}\left[2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}(t)r_{ij}(t)\left(f_{ij}(t)-\tau_n(t)\right)-\dot{\tau}(t)\right]\ge$$

$$\ge\alpha\left\{t_h+4\int_0^{t_h}\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}(t)q_{ij}(t)\left(f_{ij}(t)-\tau_n(t)\right)\right]dt\right\}-$$

$$-\left[2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}B_r\left(B_f+B_f\right)+B_{\dot{\tau}}\right]t_h\ge$$

$$\ge\left\{\alpha-8B_qB_f\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}-4B_rB_f\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}-B_{\dot{\tau}}\right\}t_h=$$

$$=\left[\alpha-\left(8B_qB_f+4B_rB_f+B_{\dot{\tau}}\right)\right]t_h \quad (29)$$

Obviously, for all times $t < t_h$, taking into account the chosen sliding mode controller gain α in (19) it follows from (26) that

$$\tau(t)\dot{\tau}(t)=-\alpha\left|\tau_c(t)\right|\left[1+4\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}q_{ij}\left(f_{ij}-\tau_n\right)\right]-$$

$$-\left[2\sum_{i=1}^{I}\sum_{j=1}^{J}\bar{W}_{ij}r_{ij}\left(f_{ij}-\tau_n\right)-\dot{\tau}\right]\tau_c(t)\le$$

$$\le\left[-\alpha\left(1-8B_qB_f\right)+4B_rB_f+B_{\dot{\tau}}\right]\left|\tau_c(t)\right|<0 \quad (30)$$

and a sliding motion exists on $\tau_c(t)=0$ for $t > t_h$.

The relation between the sliding line S_p and the zero adaptive learning error level S_c, if χ is taken as $\lambda=\frac{k_P}{k_D}$, is determined by the following equation:

$$S_c=\tau_c=k_D\dot{e}+k_Pe=k_D\left(\dot{e}+\frac{k_p}{k_D}e\right)=k_DS_p \quad (31)$$

The tracking performance of the slip control system in ABS can be analyzed by introducing the following Lyapunov function candidate:

$$V_p=\frac{1}{2}S_p^2 \quad (32)$$

Theorem 2: If the adaptation strategy for the adjustable parameters of the NFFC is chosen as in equations (16)-(18) then the negative definiteness of the time derivative of the Lyapunov function in (32) is ensured.

Proof: Evaluating the time derivative of the Lyapunov function in (27) yields:

$$\dot{V}_p=\dot{S}_pS_p=\frac{1}{k_D^2}\dot{S}_cS_c\le$$

$$\le\frac{\left|\tau_c\right|}{k_D^2}\left[-\alpha\left(1-8B_qB_f\right)+4B_rB_f+B_{\dot{\tau}}\right]<0,\ \forall S_c,S_p\ne0 \quad (33)$$

Remark: The obtained result means that, assuming the sliding mode control task is achievable, using τ_c as a learning error for the NFFC together with the adaptation laws (16)-(18) enforce the desired reaching mode followed by a sliding regime for the slip control system in ABS.

III. Simulation Results

In this section, a number of computer simulated dynamic responses are obtained to investigate the performance of the proposed control algorithm. The sampling time is $0.5ms$ and $I=J=3$ for all the simulations. All figures below show simulation results for the quarter vehicle model with initial longitudinal velocity of $V=20m/s$ maneuvering on a straight line. The reference wheel slip, which is the slip value that corresponds to the peak value of tire road friction coefficient, is calculated for different road conditions, i.e. rough ice, compact snow and dry asphalt. During the simulation studies, the vehicle is considered going out from rough ice road conditions to compact snow and then from compact snow to dry asphalt. The numerical values used for the parameters of the quarter vehicle model in this study can be seen in [6]

Fig. 3 shows the response of the system to a PID controller alone and the proposed neuro-fuzzy adaptive controller.

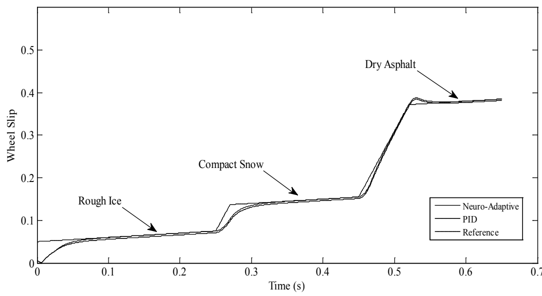

Figure 3. The wheel slip

It can be seen that the NFFC is able to learn the system dynamics in a finite time interval and to improve the system performance when compared to the usage of the PID controller alone.

The control signals for both investigated control structures are shown on Fig. 4.

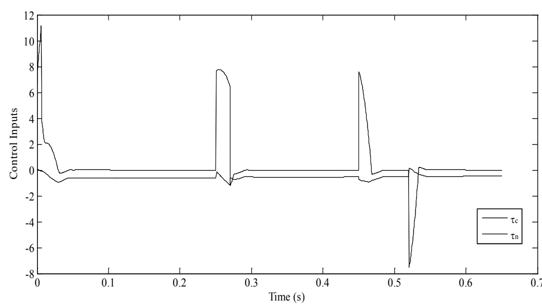

Figure 4. The control signal

It can be noticed that the proposed neuro-fuzzy adaptive controller is able to take over the control operation and thus to become the leading controller after a short time period. This results in zero output from the PD controller. Its output becomes nonzero only during the time intervals when the vehicle goes from one road condition to another.

Fig. 5 shows the phase space behavior for the first 0.22 s.

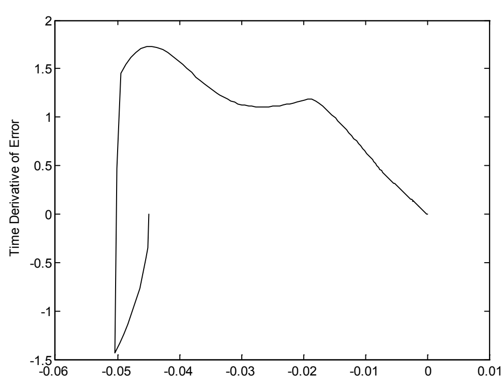

Figure 5. Phase space behavior

It figures out that $S_p = 0$ line is attracting invariant. Clearly the error vector is guided towards the sliding manifold and it is forced to remain in the vicinity of the attracting loci without explicitly knowing the analytical details of the equations of motion of the slip control system.

IV. CONLUSIONS

A novel approach for generating and maintaining sliding regime in the behavior of a system with uncertainties in its dynamics is introduced. The system under control is under a closed-loop simultaneously with a conventional PD controller and an adaptive variable structure neuro-fuzzy controller. The presented results from a simulated control of the slip in ABS have demonstrated that the predefined sliding regime could be generated and maintained if the NFFC parameters are tuned in such a way that the reaching is enforced. Another prominent feature that should be emphasized is the computational simplicity of the proposed approach.

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